Environ Resource Econ
DOI 10.1007/s10640-016-0026-2

Optimal Management of Environmental Externalities with Time Lags and Uncertainty

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Accepted: 4 May 2016

Abstract Many environmental externalities occur with time lags that can range from a few days to several centuries in length, and many of these externalities are also subject to uncertainty. In this paper, we examine the key features of an optimal policy to manage environmental externalities that are both lagged and stochastic. We develop a two-period, two-polluter model and obtain closed-form solutions for optimal emissions levels under different combinations of damage functions and stochastic processes. These solutions show that it is not obvious whether greater control should be exerted on polluters that generate externalities with longer lags or on polluters that generate externalities with shorter lags. We find that the optimal ranking of polluters with respect to the length of the time lag associated with their externality will depend on (a) the discount rate, (b) conditional expectations of future states of the polluted resource, (c) persistence of the pollutant, and (d) initial conditions.

Keywords Environmental externalities · Time lags · Uncertainty · Persistence

JEL Classification H23 · Q5 · D9 · Q2

1 Introduction

Many environmental externalities occur with time lags that can range from a few days to several centuries in length. For example, epidemiological studies have identified time lags between exposure to particulate air pollution and respiratory and cardiovascular deaths (Braga et al. 2001; Schwartz 2000). Likewise, climate change studies predict that even if greenhouse gas concentrations in the atmosphere were to stabilize today, further global warming and sea

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Published online: 19 May 2016

level rise are likely to occur until at least the year 2400 (Meehl et al. 2005; Solomon et al. 2009; Wigley 2005). Time lags have also been identified in the production of environmental benefits from pollution abatement. For example, several studies warn that improvements in water quality and ecosystem health from the restoration of streams and rivers may take decades or even centuries to materialize due to delays inherent to hydrological and biogeochemical processes (Hamilton 2012; Meals et al. 2010). However, despite their prevalence, time lags in externalities are rarely addressed in the economics literature and are often not taken into account in the design of environmental policy.

Many of these natural systems that exhibit time lags are also subject to uncertainty (Allen et al. 2000; Beck 1987; Fox 1984). Air and water pollution are inherently stochastic, with factors such as weather playing a causal role in these processes. Uncertainty may also stem from the inability of regulators to monitor these natural systems on a continuous and widespread basis at reasonable cost. While a large number of studies have explored how environmental policies can optimally address uncertainty, much of this literature has focused on the asymmetry between price and quantity instruments in the presence of uncertainty concerning the costs of pollution control (Weitzman 1974), the implications of irreversibilities in environmental damage (Pindyck 2002), and the role of regime shifts in defining optimal precautionary behavior (Brozović and Schlenker 2011; Zemel 2012). These studies do not explicitly address how time lags affect the optimal management of these stochastic natural systems.

In this paper, we examine the key features of an optimal policy to manage environmental externalities that are both lagged and stochastic. We develop a two-period, two-polluter model that allows us to obtain closed-form solutions for optimal emissions levels under different combinations of environmental damage functions and stochastic processes. Despite the relative simplicity of the model, we find that the simultaneous presence of lags and uncertainty leads to an optimal policy with differentiated regulation of polluters defined by a complex interaction between the effects of discounting, persistence of the pollutant, and initial conditions. In addition, the closed-form solutions yield analytical results regarding the relationship between optimal emissions levels and key parameters, thus helping us identify the specific mechanisms through which lags and uncertainty affect optimal pollution policies. The fact that such an elaborate optimal emissions policy arises from our two-period, two-polluter model suggests that policymakers should exercise care in regulating lagged environmental externalities in the real world, which are likely to be more complicated.

Our analysis makes two main contributions to the literature on the management of environmental externalities. First, we develop a model and derive solutions for the optimal management of environmental externalities when heterogeneous time lags and uncertainty are simultaneously present in a setting with multiple polluters. To our knowledge, no previous studies have described how lags and uncertainty affect the tradeoff between emissions by different polluters in achieving a socially optimal level of aggregate environmental damage. We thus show that accounting for these common characteristics of externalities may be important from a policy perspective, while the derivation of closed-form solutions for the optimal emissions policy allows us to understand how policies would need to be adjusted in response to changes in key parameters such as the discount rate, initial conditions, and the degree of uncertainty associated with the stochastic process.

Our second contribution is to provide new results that contradict previous theoretical findings in the environmental regulation literature. While previous studies have concluded that introducing longer time lags into an environmental externality increases the optimal level of the activity that generates that externality (Fleming et al. 1995; Kim et al. 1993), we show that it may be optimal for regulators to allow greater levels of the externality with shorter time lags relative to the externality with longer time lags. This difference in our results arises from the fact that the aforementioned studies employed deterministic models, whereas our models are stochastic. Our results also differ from those commonly found in the vast literature on water quality markets for point and non-point source pollution (see for example, Horan 2001; Malik et al. 1993; Shortle 1987). These studies suggest that trading policies in these markets should be designed so as to encourage greater control of non-point sources relative to point sources due to the larger uncertainty associated with emissions from non-point sources. In contrast, our results indicate that when a regulator allocates emissions levels to two polluters that generate lagged externalities, it may be optimal to encourage greater emissions from the polluter associated with greater uncertainty. We find that such an optimal allocation of emissions can arise if the polluter associated with greater uncertainty is also associated with environmental damages that occur with very long time lags.

Our paper proceeds as follows. Section 2 provides a brief summary of existing studies that address the role of time lags in optimal economic decision-making. We present our two-period, two-polluter model in Sect. 3, and in Sect. 4 we investigate the properties of the optimal emissions policy that results from the model. Section 5 provides a discussion of the policy implications of our results, and Sect. 6 concludes.

2 Background: Time Lags and Environmental Policy

In our analysis, we will focus on the role of time lags in physical processes of environmental systems as opposed to lags in socioeconomic systems, such as those that arise from regulatory processes or human behavior. In many cases, lags in physical processes are caused by spatial processes in which it takes a certain amount of time for a contaminant to be transported from its source to the location where it causes environmental damage. For example, if contaminants are released in a river upstream from a drinking water source, it may take several days or weeks for the contaminants to flow down the river and affect drinking water quality. Similarly, there is evidence that current patterns of invasive species presence better reflect historical rather than contemporary human activities (Essl et al. 2001). In other cases, lags may be caused by inertia in physical or chemical processes such that a natural system does not respond instantaneously to increases or decreases in the emissions of a pollutant. For example, even if greenhouse gas concentrations in the Earth's atmosphere were to be reduced in the short term, because it will take some time for the oceans to dissipate the heat that they have already taken up, desired reductions in temperature on Earth may take much longer to materialize. More generally, we are interested in all environmental externalities in which the negative effects of an economic activity do not take place instantaneously but instead take place in some time period in the future.

Moreover, we will focus on situations in which a regulator must manage multiple polluters that negatively affect the environment with different time lags. While polluters that generate externalities with different time lags could be regulated separately, a social planner may wish to regulate them jointly if the pollution affects the same environmental medium. By regulating these polluters jointly, the social planner will be able to account for tradeoffs between the emissions of the different polluters so as to achieve a desired reduction in pollution at minimum aggregate cost. A concrete example of such a scenario would be the comprehensive regulation of air pollutants that contribute to climate change, in which a social planner would seek an optimal allocation of emissions of carbon dioxide, methane, black carbon, and other contaminants. Our analysis will show that the optimal allocation of emissions across these different pollutants could be driven in part by the fact that increases or decreases in carbon dioxide emissions will take longer to affect global temperatures compared to changes in methane and black carbon emissions (UNEP 2011). Another real-world example is that of a water resource manager who is concerned about pollution in a target water body due to nitrates delivered in groundwater discharge from agricultural fields and animal operations. In this situation, the manager will need to account for lags between implementation of best management practices and water quality improvements in the target water body that range from 1 to 100 years, depending on the location of the pollutant source (Meals and Dressing 2008).

Previous studies on the impact of time lags on optimal economic decision-making have largely focused on problems outside the realm of environmental and natural resource economics, primarily on cases in which current decisions by a firm impact the demand for their products with a lag. These studies include models of dynamic advertising policy (Kamien and Muller 1976; Nerlove and Arrow 1962), population accumulation and its implications on the macroeconomy (Arthur and McNicoll 1977; van Imhoff 1989), and the production and taxation of durable goods (Muller and Peles 1988, 1990; Goering and Boyce 1999; Runkel 2003). In these studies, lags are shown to be a significant determinant of the profit-maximizing strategy of individual firms. However, the conclusions provided by these models are not directly applicable to environmental issues, since lags in these applications only impact private costs and thus do not affect the rest of society through externalities.

In the environmental economics literature, a series of papers has incorporated time lags in models of optimal control of groundwater pollution. In modeling the response of groundwater systems to emissions, these studies adopt a stock-and-flow approach similar to the state equations that describe capital accumulation in the macroeconomics literature. In most cases, emissions are treated as a flow that, with some temporal delay, gets added to a stock of ambient pollution that decays over time. Kim et al. (1993) and Fleming et al. (1995) develop dynamic models that examine the effect of time lags between nitrogenous fertilizer application and nitrate contamination of aquifers on optimal regulatory policy. Conrad and Olson (1992), Yadav (1997), Nkonya and Featherstone (2000) and Ibendahl and Fleming (2007) conduct empirical studies of agricultural practices in the US and use the estimated models to test the effect of regulations such as nitrogen standards and user fees. Recent work has also examined the role of lags in optimal carbon sequestration paths (Ragot and Schubert 2008; Caparrós 2009).

While these studies provide compelling theoretical and empirical evidence that lags play a key role in determining optimal economic activity in the presence of externalities, models employed in these studies only involve one decision-maker. As a result, an optimal policy does not have to adjust for the tradeoffs involved in a regulatory context with multiple, heterogeneous agents. We explore these tradeoffs in two of the four model formulations we develop below, namely, those for which emissions from firms can interact through the stock of accumulated pollution. Another difference between our analysis and existing studies is that the models we present are stochastic, allowing us to account for the possible interaction between lags and uncertainty in the decision-maker's problem.

3 A Model with Two Firms and Two Time Periods

In this section, we present a social planner's problem in which a regulator must assign optimal emissions levels to two polluters, one of which causes environmental damages immediately while the other causes environmental damages with a one-period lag. Because we are interested in time lags in physical processes of environmental systems as opposed to lags in socioeconomic systems, we assume that the structure of the lags are determined exogenously and cannot be modified by the regulator or by the polluters. We will choose functional forms to represent the polluters' abatement costs, the environmental damages caused by pollution, and the stochastic process that affects pollution over time. In developing these components, our goal is to present the simplest model that incorporates lags and uncertainty that will still yield concrete insights regarding an optimal emissions policy. While real-world environmental problems involving lagged externalities are likely to be more complicated than the model we present here, we will show that even a relatively simple setting with lags and uncertainty will imply optimal emissions strategies that are qualitatively different from those that arise from models that account for only lags or only uncertainty. Furthermore, our use of explicit functional forms and derivation of closed-form solutions provide clarity and allow us to identify the specific economic forces driving the optimal management strategy.

3.1 Abatement Costs and Environmental Damages

Consider a market consisting of two polluters, Firms A and B. Both firms generate emissions in the first time period only and do not generate emissions in the second period. In the absence of regulation, both Firms A and B each choose to emit at a level given by $\bar{e}$. However, in the presence of regulation, each firm may choose to reduce its emissions to a level e_j, where $j = A, B$ and $e_j < \bar{e}$, at a cost that is given by identical linear abatement cost functions:

$$C_j(\bar{e}_j - e_j) = \phi(\bar{e} - e_j) \quad \text{for} \quad j = A, B. \tag{1}$$

In Eq. (1), $\bar{e}_j - e_j$ represents the quantity of emissions abated by firm j and ϕ is a scaling parameter. This functional form implies that firms can incrementally reduce their emissions at constant marginal cost.[1] We choose identical abatement cost functions for the two firms for ease of exposition and in order to focus our analysis on the role of time lags and uncertainty on optimal emission rates; assigning different abatement cost functions to the two firms does not lead to qualitative differences in our results.

3.2 Lags

Although the two firms generate emissions only in the first period, these emissions can affect the environment in the first period as well as in the second period due to the presence of time lags. In our model, emissions by the two firms pollute a single natural resource or environmental medium, the state of which is represented by the variable x_t, for $t = 1, 2$. As such, the state variable x_t can represent the concentration of pollutants measured at an air quality monitoring station or the accumulation of carcinogens in a stream or aquifer in time period t. Hereafter, we will refer to x_t as the *ambient contaminant concentration* at time t.

Emissions by the two firms affect ambient contaminant concentration differently over the two time periods in our model. Specifically, while both Firms A and B generate emissions in time period $t = 1$, Firm A's emissions get added to ambient contaminant concentration in period $t = 1$, while Firm B's emissions get added to ambient contaminant concentration in period $t = 2$. In other words, emissions by Firm A generate environmental damages

[1] We are unable to derive closed-form solutions for optimal emissions levels when firms' abatement costs functions are nonlinear. However, we are able to show that the relationship between key model parameters and optimal emissions levels are largely the same under linear or nonlinear cost functions. Please refer to "Optimal Emission Levels Under Generic Abatement Costs" section in "Appendix" for these derivations.

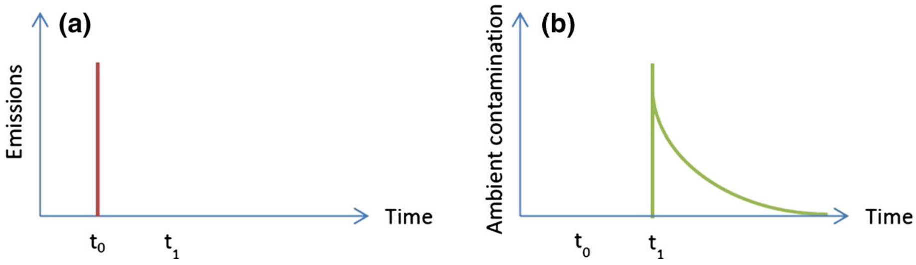

Fig. 1 Graphical illustration of the difference between time lags and persistence

instantaneously, while emissions by Firm B generate environmental damages with a one-period lag. As an illustrative example, consider a situation in which a social planner cares about the impacts of air pollutants on global mean temperature. Firm A could be a polluter that primarily emits black carbon, such that temperatures are affected almost immediately by emissions by that firm. In contrast, Firm B could be a polluter that primarily emits carbon dioxide, and as a result, temperatures are affected by emissions with a much longer delay.

We will allow for persistence in ambient contaminant concentrations over time, that is, we will allow concentrations in time period 2 to be determined in part by concentrations in time period 1. The degree of persistence will be represented by the parameter ρ in our model. For our analysis, it is important to distinguish the effects of time lags from those of persistence. When the environmental impacts of a pollutant are persistent but not lagged, emissions of the pollutant start generating impacts immediately and continue to do so in future time periods even after emissions have ceased. In contrast, when time lags are present, environmental impacts don't start occurring until a certain amount of time after the emissions take place. Figure 1 provides a graphical illustration of the difference between time lags and persistence. Panel (a) depicts a situation in which a unit of emissions takes place at time t_0 without any further emissions after t_0. Panel (b) illustrates the impact of this unit of emissions on ambient contaminant concentrations over time when the pollutant is characterized by both time lags and persistence. The presence of a time lag implies that ambient contaminant concentrations are not affected by emissions until some time after t_0. Persistence of the pollutant implies that once the pollutants start affecting ambient contaminant concentrations, they continue to do so for a while. The figure helps illustrate the subtle but important difference between these two dynamic effects; they are modeled separately in our analysis, and our results show that they have qualitatively different effects on optimal emissions policies.

3.3 Uncertainty

In addition to being affected by firm emissions, ambient contaminant concentration is subject to a stochastic shock θ_t in each time period. Although the realized shock in time period 1 is observable, the shock in time period 2 is unknown to firms or to the social planner. Examples of stochastic processes that affect ambient contaminant concentrations in the real world include wind speed and direction, precipitation, and temperature. The stochastic shock θ_t can also represent uncertainty that results from insufficient knowledge regarding the true behavior of natural systems, such as uncertainty over the future impacts of climate change.

Because different environmental systems may be subject to different types of stochastic processes, we will consider two alternative forms of uncertainty in the evolution of ambient contaminant concentration over time. While both of the stochastic processes we consider are relatively simple, our analysis will show that even small differences in the type of uncertainty associated with an environmental system will lead to significantly different optimal strategies to manage lagged externalities that affect that system.

3.3.1 Uncertainty Type 1: Serial Correlation in the Shocks to Ambient Contaminant Concentrations

For the first type of uncertainty, we allow no serial correlation in ambient contaminant concentration (i.e. $\rho = 0$) but do allow for serial correlation in the shocks to ambient contaminant concentrations, θ_t. The evolution of state variable x_t can be expressed as:

$$x_t = \begin{cases} e_A + \theta_1 & \text{if } t = 1, \\ e_B + \theta_2 & \text{if } t = 2, \end{cases}$$

where $\theta_2 = \mu\theta_1 + \epsilon_2$, $\epsilon_2 \sim N(0, \sigma_\epsilon^2)$, and θ_1 is given. The parameter $\mu \in (0, 1)$ describes the persistence of the shocks to ambient contaminant concentration. More generally, this form of uncertainty is consistent with shocks to environmental systems that are correlated over time, so that positive shocks are more likely to be followed by positive shocks rather than negative shocks, and vice-versa. At the same time, in this form of uncertainty, the state of the environment itself in any time period is independent of the state of the environment in other time periods.

One real-world scenario that fits the above description is a local air pollutant that dissipates relatively quickly after it is emitted, such as particulate matter. Thanks to the rapid dissipation of the pollutant, ambient contaminant concentrations return to zero after one time period, unless additional emissions are released. Serially correlated shocks in this scenario could be generated by temperature fluctuations, where warm days are more likely to be followed by warm days rather than cold days, and vice versa. Temperature is often an important determinant of ambient air quality but is also difficult to predict far in advance (Camalier et al. 2007).

It is important to note that, under this first type of uncertainty, the influence of emissions by the two firms on ambient contaminant concentration is isolated across the two time periods; that is, ambient contaminant concentration in time period 1 is only affected by Firm A, and ambient contaminant concentration in time period 2 is only affected by Firm B. As a result, pollution by the two firms does not interact under uncertainty type 1, and the social planner's problem is equivalent to solving for each firm's optimal emissions rate separately. Uncertainty type 2, which we describe below, allows both Firms A and B to affect ambient contaminant concentrations in the same time period, leading to an interaction between the two polluters.

3.3.2 Uncertainty Type 2: Serial Correlation in Ambient Contaminant Concentrations

In the second form of uncertainty, we allow for serial correlation in ambient contaminant concentration (i.e. $\rho > 0$) but no serial correlation in the shocks θ_t. Under this form of uncertainty, the state equation can be written as an AR(1) process as follows:

$$x_t = \begin{cases} \rho x_0 + e_A + \theta_1 & \text{if } t = 1, \\ \rho x_1 + e_B + \theta_2 & \text{if } t = 2, \end{cases}$$

where $\theta_2 \sim N(0, \sigma_\theta^2)$, and x_0 as well as θ_1 are given. As a result, the level of ambient contaminant concentration in period $t = 2$ is a function of the level of ambient contaminant concentration in period $t = 1$, but the shock to ambient contaminant concentration in period $t = 2$ is independent of the shock in period $t = 1$. This second form of uncertainty is consistent with the behavior of environmental systems that exhibit persistence over time. For example, this process could describe the concentration of a pollutant that accumulates in the environment because the rate at which it is injected into the environment exceeds the environment's capacity to assimilate the pollutant.[2]

In this second form of uncertainty, emissions by both Firms A and B influence ambient contaminant concentration in time period 2. This is in contrast to pollution dynamics under uncertainty type 1, in which emissions by the firms do not interact. The fact that there is an interaction between polluters in uncertainty type 2 will introduce a tradeoff between emissions that generate immediate and delayed damages.

Under both uncertainty type 1 and 2, the social planner can reduce the risk of future environmental damages by decreasing the amount of pollutants emitted in the future. However, our model does not allow the social planner to reduce the probability of experiencing a negative shock in the future. Using the terminology of Ehrlich and Becker (1972), the social planner in our models is able to self-insure, but not able to self-protect, against future uncertainty by forgoing current emissions. An interesting extension to our model would allow for the social planner to take actions that adjust the probability distribution of shocks to ambient contaminant concentration, as such strategies may be relevant to real-world environmental management problems.

We can also draw parallels between our formulations of uncertainty and the analysis of Shogren and Crocker (1991, 1999), who distinguish between exogenous risk, which is involuntary exposure to risk, and endogenous risk, which is voluntary exposure to risk. Uncertainty types 1 and 2 can be described as involving exogenous and endogenous risk, respectively, because in the latter form of uncertainty, the social planner can self-insure against future uncertainty by forgoing current emissions by Firm A.

3.4 Environmental Damage Function

Positive levels of ambient contaminant concentration lead to damages in the environment, the social costs of which are represented by a damage function. Examples of such social costs include negative impacts to human health, reduced availability of clean water, or a loss in biodiversity. In our analysis, we will choose two different formulations for the environmental damage function: quadratic functions and exponential functions. Quadratic and exponential functional forms have been used extensively in the literature to model environmental damages from pollution (see, for example, Conrad and López 2002; Harper and Zilberman 1992; Laukkanen and Huhtala 2008; Weitzman 2010). As we will show below, these two functional forms will lead to different optimal emissions policies based on how they relate uncertainty to expected marginal damages.

[2] Because both the initial ambient contaminant concentration level (x_0) and the shock in time period 1 (θ_1) are additive, it is possible to choose a different notation that represents the two initial conditions using only one variable. In fact, our analysis will show that changes in x_0 and θ_1 have the same qualitative impact on optimal emissions. However, we have decided to retain the notation that separates the initial stock level from the initial shock in order to better conform to the literature, which often uses recursive formulations to characterize the evolution of stocks over time.

3.4.1 Quadratic Damage Function

The first formulation makes environmental damage costs quadratic in the level of the pollution stock:

$$D(x_t) = \alpha x_t + \frac{\gamma}{2} x_t^2 \quad \text{for} \quad t = 1, 2, \tag{2}$$

where α and γ are scaling parameters for the damage function. Together, α and γ determine how rapidly environmental damages increase as a result of higher ambient contaminant concentration. Note that with a quadratic damage function, marginal damages are linear in the level of ambient contaminant concentration.

3.4.2 Exponential Damage Function

The second environmental damage function we consider takes on an exponential form:

$$D(x_t) = \exp(\psi x_t) - 1 \quad \text{for} \quad t = 1, 2, \tag{3}$$

where ψ is a scaling parameter. Unlike the quadratic damage function, marginal damages are convex in the level of ambient contaminant concentration in this second damage function specification. Note that ψ is the damage function-equivalent of the coefficient of risk aversion for exponential utility functions in consumer theory. In our context, ψ can be thought of as representing the degree to which uncertainty in environmental damage outcomes is costly to society. However, ψ should not be thought of as a measure of risk preference, because unlike utility functions, damage functions only represent monetized physical damages caused by pollution.

4 Properties of the Optimal Emissions Policy

In this section, we will combine the two formulations of the environmental damage function and the two types of uncertainty defined above in order to develop four different social planner's problems. The social planner's problems have the following general form:

$$\min_{e_A, e_B} \quad \phi(\bar{e} - e_A) + \phi(\bar{e} - e_B) + D(x_1) + \beta \mathbb{E}\left[D(x_2) | \theta_1 \right], \tag{4}$$

where the damage functions $D(\cdot)$ take on quadratic (Eq. 2) or exponential (Eq. 3) forms and the stochastic ambient contaminant concentration levels x_1 and x_2 follow uncertainty type 1 (Sect. 3.3.1) or uncertainty type 2 (Sect. 3.3.2). In combination, this yields social planner's problems involving (a) quadratic damages with uncertainty type 1, (b) exponential damages with uncertainty type 1, (c) quadratic damages with uncertainty type 2, and (d) exponential damages with uncertainty type 2. For each social planner's problem, we will derive closed-form solutions for optimal emissions levels for Firms A and B. We will also explore how these optimal emissions levels depend on the magnitude of key modeling parameters, namely, the discount factor (β), initial conditions for ambient contaminant concentration (θ_1 and x_0), indicators of persistence (μ and ρ) and the variance of the stochastic process affecting ambient contaminant concentration (σ_ϵ^2 and σ_θ^2).

4.1 Quadratic Damages and Uncertainty Type 1

Let e_A^* and e_B^* denote optimal emissions levels for Firms A and B, respectively. When the structure of uncertainty is such that there is serial correlation in the shocks to ambient contaminant concentration but no serial correlation in ambient contaminant concentration per se, solving the minimization problem in (4) leads to the following result:

Proposition 1 *Under a quadratic damage function and uncertainty type* 1*, optimal emissions levels for Firms A and B are equal to:*

$$e_A^* = \frac{\phi - \alpha}{\gamma} - \theta_1, \tag{5}$$

$$e_B^* = \frac{1}{\gamma}\left(\frac{\phi}{\beta} - \alpha\right) - \mu\theta_1. \tag{6}$$

Proof See derivation in "Optimal Emission Levels Under Uncertainty Type 1 and Quadratic Damages" section in "Appendix". □

According to Eqs. (5) and (6), under a quadratic damage function and uncertainty type 1, the optimal emissions level for Firm A is decreasing in the level of the initial shock to ambient contaminant concentration, θ_1, but is independent of the discount factor, β, the degree of persistence of shocks to ambient contaminant concentration, μ, and the variance in shocks to ambient contaminant concentration, σ_ϵ^2. The optimal emissions level for Firm B is also decreasing in θ_1 and independent of σ_ϵ^2, but unlike the optimal emissions level for Firm A, it is decreasing in β. In addition, the relationship between e_B^* and μ depends on the sign of the initial shock θ_1. If θ_1 is positive (i.e. the initial shock is unfavorable to ambient contaminant concentrations), greater persistence is associated with lower optimal emissions by Firm B, and vice versa.[3]

The intuition behind the relationship between optimal emissions for the two firms and β is relatively straightforward. Because damages caused by Firm A's emissions occur instantaneously but damages caused by Firm B's emissions occur in the future, the social costs of Firm B's emissions are discounted but those of Firm A are not. As a result, for a higher level of the discount factor β (which is equivalent to a lower level of the discount rate), it is optimal for the social planner to allow more emissions from Firm A than from Firm B. Equations (5) and (6) also show that the difference in optimal emissions levels for Firms A and B gets larger as the discount factor decreases (that is, as the discount rate increases).

Another result associated with Proposition 1 is that optimal emissions levels for both firms are decreasing in the size of the initial shock to ambient contaminant concentrations, θ_1, which is observed at the time when the social planner chooses the emissions levels. Because the damage function is convex in ambient contaminant concentration, if the social planner observes an unfavorable shock ($\theta_1 > 0$), her optimal strategy is to choose a lower level of emissions for Firm A relative to the case in which there is no shock. Furthermore, since shocks are correlated over time under uncertainty type 1, the expected shock in the second time period, θ_2, conditional on observing a shock θ_1 in the first time period, is equal to $\mathbb{E}[\theta_2|\theta_1] = \mu\theta_1$. Thus, observing an unfavorable initial shock also leads to a lower optimal emissions level for Firm B, even though its emissions only affect ambient contaminant concentration in the second time period. Note that the smaller the magnitude of the persistence

[3] Note that in our model, positive (i.e. >0) realizations of the shocks θ_1 and θ_2 are actually "bad" from society's point of view because they lead to greater environmental damage.

parameter μ, the smaller the adjustment that is made to Firm B's optimal emissions in response to the observed initial shock.

In addition, because $|\theta_1| > |\mu\theta_1|$, larger, unfavorable initial shocks to ambient contaminant concentration lead to greater reductions in emissions by Firm A than by Firm B; that is, the optimal policy will favor emissions that generate environmental damages with longer lags. Intuitively, if a social planner sees a large unfavorable shock in the first time period, she will try to move the environmental damages from emissions farther into the future, since the conditional expected mean of the shock in the future is smaller in magnitude than the shock that is observed in the present. Conversely, if the initial shock to the resource is favorable, the social planner will adjust the optimal emissions rates for both firms upwards, with a larger increase for Firm A than for Firm B, thus bringing environmental damages relatively closer to the present.

Comparative statics also state that the variance in shocks to ambient contaminant concentration, σ_ϵ^2, has no effect on the optimal emissions level of either firm. This result is due to the fact that under a quadratic damage function, marginal damages are linear (i.e. the third derivative of the damage function is equal to zero). Therefore, an increase in the variance of ambient contaminant concentration has no effect on expected marginal damages, and thus has no effect on optimal emissions rates.

The last key result associated with Proposition 1 is that the optimal emissions level for Firm A may be greater than or less than the optimal emissions level for Firm B. The reason behind this ambiguity can be established by deriving the expression for the difference in optimal emissions rates:

$$e_A^* - e_B^* = \frac{\phi}{\gamma}\left(1 - \frac{1}{\beta}\right) + \theta_1(\mu - 1). \tag{7}$$

Because $\beta \in (0, 1)$, the first group of terms on the right-hand side of Eq. (7) is always negative, thus implying higher emissions levels for Firm B than for Firm A ($e_A^* < e_B^*$). However, assuming that $\mu \in (0, 1)$, if the social planner observes a sufficiently large negative value of θ_1, optimal emissions may be higher for Firm A than for Firm B ($e_A^* > e_B^*$). In other words, while discounting will encourage the social planner to adjust emissions by the two firms so as to push environmental damages farther into the future, if a sufficiently favorable shock is observed in the first period, it will be optimal for the social planner to try to move damages closer to the present due to the convexity of the damage function. As a result, the relative sizes of the "discounting effect" and the "initial shock effect" will determine the ranking of the two firms with respect to their optimal emissions rates.

4.2 Exponential Damages and Uncertainty Type 1

We now show how the relationships between key parameters and optimal emissions change when the environmental damage function has an exponential form instead of a quadratic form:

Proposition 2 *Under an exponential damage function and uncertainty type* 1*, optimal emissions levels for Firms A and B are equal to:*

$$e_A^* = \frac{1}{\psi}\ln\left(\frac{\phi}{\psi}\right) - \theta_1, \tag{8}$$

$$e_B^* = \frac{1}{\psi}\ln\left(\frac{\phi}{\psi\beta}\right) - \mu\theta_1 - \frac{\psi\sigma_\epsilon^2}{2}. \tag{9}$$

Proof See derivation in "Optimal Emission Levels Under Uncertainty Type 1 and Exponential Damages" section in "Appendix". □

According to Eqs. (8) and (9), under an exponential damage function and uncertainty type 1, the optimal emissions level by Firm A exhibits the same relationship with key modeling parameters as in the case with a quadratic damage function; optimal Firm A emissions are decreasing in θ_1 but independent of β, μ, and σ_ϵ^2. Likewise, optimal Firm B emissions are decreasing in β and θ_1, while the effect of μ depends on the sign of the initial shock θ_1. However, unlike the case with a quadratic damage function, under an exponential damage function, optimal Firm B emissions are decreasing in σ_ϵ^2. While changes in σ_ϵ^2 had no effect on optimal emissions levels under quadratic damages, under exponential damages, increases in σ_ϵ^2 are associated with lower levels of optimal emissions for Firm B, the firm with longer lags in damages.

This inverse relationship between Firm B optimal emissions and the degree of uncertainty is driven by two factors. The first is that, under uncertainty type 1, the conditional variance of the stochastic process increases the farther the social planner looks into the future. In our two-period model, this is embodied in the fact that the shock in the first time period is observed (and thus carries no uncertainty) and the shock in the second time period is associated with a level of variance σ_ϵ^2.[4] As a result, the social planner's ability to forecast ambient contaminant concentrations degrades over time.

The second factor that drives the inverse relationship between Firm B optimal emissions and the degree of uncertainty is the curvature of the exponential damage function, which is characterized by a positive third derivative. This means that marginal damages are convex in ambient contaminant concentration, and an increase in uncertainty raises expected marginal damages. At the same time, under uncertainty type 1, society's uncertainty regarding the state of the resource increases the farther it tries to look into the future. As a result, in an optimal solution, expected future ambient contaminant concentrations must decrease relative to current ambient contaminant concentrations, which is accomplished by reducing emissions by Firm B. More intuitively, in the presence of an exponential damage function, the social planner has an incentive to bring expected environmental damages closer to the present, into a period of time during which the level of ambient contaminant concentration is more predictable.

This reallocation of environmental damages over time in response to increased uncertainty is analogous to consumer behavior in models of precautionary savings from the macroeconomic literature (Leland 1968; Sandmo 1970). In these models, increases in the variance of consumption leads consumers to be more "prudent" by sacrificing current consumption in favor of future consumption, as long as the third derivative of their utility functions is positive. However, as noted previously, we must exercise care when employing the analogy of consumer behavior when describing the solution to our model. The fact that optimal emissions depend on the degree of uncertainty in pollution should not be interpreted as being an outcome of risk preferences. In fact, the social planner in our model is risk neutral, but the degree of uncertainty is relevant in determining optimal emissions because of the convex

[4] In a model with more than two time periods, the conditional variance of the stochastic process in any given period s would be equal to:

$$\mathrm{Var}[\theta_s|\theta_1] = \sigma_\epsilon^2 \sum_{i=0}^{s-1} \mu^{2i}, \tag{10}$$

which increases monotonically over time as it converges to the unconditional variance, $\sigma_\epsilon^2/(1-\mu^2)$.

relationship between marginal damages and ambient contaminant concentrations under an exponential damage function.

As in the quadratic damage function case, optimal emissions by Firm A may be higher than or lower than optimal emissions by Firm B under an exponential damage function. The difference between optimal emissions levels in this case is given by:

$$e_A^* - e_B^* = \frac{1}{\psi}\ln(\beta) + \theta_1(\mu - 1) + \frac{1}{2}\psi\sigma_\epsilon^2. \tag{11}$$

The first two terms on the right-hand side of Eq. (11) embody the discounting effect and initial shock effect that also exist under quadratic damages. The additional third term represents the "precautionary effect" described above and contributes to the ambiguity in the optimal allocation of emissions.

4.3 Quadratic Damages and Uncertainty Type 2

We now consider optimal solutions when the social planner faces uncertainty type 2, in which ambient contaminant concentration is correlated over time but shocks to ambient contaminant concentration are not. In this case, solving the minimization problem in (4) leads to the following result:

Proposition 3 *Under a quadratic damage function and uncertainty type* 2*, optimal emissions by Firms A and B are equal to:*

$$e_A^* = \frac{\phi - \phi\rho - \alpha}{\gamma} - \rho x_0 - \theta_1, \tag{12}$$

$$e_B^* = \frac{\phi - \alpha\beta - \beta\rho(\phi - \phi\rho - \alpha)}{\gamma\beta}. \tag{13}$$

Proof See derivation in "Optimal Emission Levels Under Uncertainty Type 2 and Quadratic Damages" section in "Appendix". □

According to Eqs. (12) and (13), under a quadratic damage function and uncertainty type 2, the optimal emissions level by Firm A is decreasing in the level of the initial shock to ambient contaminant concentration, θ_1, the initial ambient contaminant concentration level, x_0, and the degree of persistence of ambient contaminant concentration, ρ, but is independent of the discount factor, β, and the variance in shocks to ambient contaminant concentration, σ_θ^2. In contrast, optimal Firm B emissions are decreasing in β but independent of θ_1, x_0, and σ_θ^2. We also find that the direction of the relationship between optimal Firm B emissions and ρ is ambiguous.

Because ambient contaminant concentration exhibits persistence over time under uncertainty type 2, the initial concentration level x_0 plays a role in determining the optimal emissions policy. Specifically, if the social planner observes a higher initial concentration level when she solves her optimization problem, she will choose a lower optimal emissions level for Firm A. This is due to the fact that choosing a lower emissions level for Firm A will lead to a lower level of marginal damages in both the first and second time periods. The social planner will also choose a lower optimal emissions level for Firm A if the observed initial shock θ_1 is unfavorable, in order to lower the level of marginal damages experienced in the first period. In this two-period model, optimal emissions by Firm B are independent of the initial level of ambient contaminant concentration as well as the initial shock to ambient contaminant concentration because (a) shocks to ambient contaminant concentrations are not correlated over time and (b) the concentrations that persist into the second time period can only be controlled by emissions in the first period. Finally, the variance parameter σ_θ^2 does not play a role in determining the optimal emissions level for either firm, for the same reason that the degree of uncertainty does not determine optimal emissions under uncertainty type 1 when the damage function is quadratic.

The difference in optimal emission rates for Firms A and B can be expressed as:

$$e_A^* - e_B^* = \frac{\phi(\beta - 1) - \beta\phi\rho^2 - \beta\rho\alpha}{\gamma\beta} - \rho x_0 - \theta_1. \tag{14}$$

Equation (14) demonstrates that, as was the case under uncertainty type 1, the initial shock effect can dominate the discounting effect so as to make the ranking of e_A^* and e_B^* ambiguous.

4.4 Exponential Damages and Uncertainty Type 2

In the fourth and final social planner's problem that we solve, we consider an exponential damage function coupled with uncertainty type 2. Solving this minimization problem leads to the following result:

Proposition 4 *Under an exponential damage function and uncertainty type* 2*, optimal emissions by Firms A and B are equal to:*

$$e_A^* = \frac{1}{\psi} \ln\left(\frac{\phi - \phi\rho}{\psi}\right) - \rho x_0 - \theta_1, \tag{15}$$

$$e_B^* = \frac{1}{\psi}\left[\ln\left(\frac{\phi}{\beta\psi}\right) - \rho \ln\left(\frac{\phi - \phi\rho}{\psi}\right)\right] - \frac{\psi\sigma_\theta^2}{2}. \tag{16}$$

Proof See derivation of optimal emissions rates in "Optimal Emission Levels Under Uncertainty Type 2 and Exponential Damages" section in "Appendix". □

According to Eqs. (15) and (16), under an exponential damage function and uncertainty type 2, the optimal emissions level by Firm A exhibits the same relationship with key modeling parameters as in the case with a quadratic damage function; optimal Firm A emissions are decreasing in θ_1, x_0, and ρ, but are independent of β and σ_θ^2. Furthermore, as was the case with the quadratic damage function, optimal Firm B emissions are decreasing in β and independent of θ_1 and x_0, and the direction of the relationship with ρ is ambiguous. However, unlike the case with the quadratic damage function, optimal Firm B emissions are decreasing in σ_θ^2, the variance in shocks to ambient contaminant concentrations.

As was the case with the exponential damage function and uncertainty type 1, the social planner has an incentive to reduce ambient contaminant concentrations during time periods associated with greater uncertainty. While the conditional variance of shocks to ambient contaminant concentration is constant over time under uncertainty type 2, the conditional variance of ambient contaminant concentration levels increases over time because of serial correlation in the concentrations.

Finally, as in all previous social planner's problems examined in this analysis, the optimal emissions level for Firm A may be greater than or less than the optimal emissions level for Firm B due to the combined effects of discounting, the initial shock to ambient contaminant concentrations, and "precaution" in the face of uncertainty. This ambiguity is evident in the expression for the difference in optimal emission rates:

$$e_A^* - e_B^* = \frac{1}{\psi}\left[(1+\rho)\ln\left(\frac{\phi - \phi\rho}{\psi}\right) - \ln\left(\frac{\phi}{\beta\psi}\right)\right] - \rho x_0 - \theta_1 + \frac{1}{2}\psi\sigma_\theta^2. \tag{17}$$

Table 1 Influence of key parameters on optimal emissions

	Quadratic damages		Exponential damages	
	Firm A (e_A^*)	Firm B (e_B^*)	Firm A (e_A^*)	Firm B (e_B^*)
Uncertainty type 1				
Discount factor (β)	0	–	0	–
Initial shock (θ_1)	–	–	–	–
Persistence in shocks (μ)	0	−, 0, or +	0	−, 0, or +
Variance (σ_ϵ^2)	0	0	0	–
Uncertainty type 2				
Discount factor (β)	0	–	0	–
Initial shock (θ_1)	–	0	–	0
Initial state (x_0)	–	0	–	0
Persistence in concentration (ρ)	–	−, 0, or +	–	−, 0, or +
Variance (σ_θ^2)	0	0	0	–

In this table, a plus sign (+) indicates a direct relationship between optimal emissions and the key modeling parameter, and a minus sign (−) indicates an inverse relationship. A zero (0) indicates a lack of relationship between optimal emission and the key modeling parameter

Table 1 summarizes the results from the four social planner's problems considered in this section and provides an overview of how the relationships between optimal emissions and key modeling parameters differ across environmental damage functions and stochastic processes.

5 Discussion

The main policy implication from the preceding analysis is that the relationship between optimal emissions and the length of the time lag associated with a polluter's externality is not straightforward in the presence of uncertainty. Under the four combinations of environmental damage functions and stochastic processes examined in our two-firm, two-period framework, the optimal emissions level of a firm generating externalities with short lags may be higher than or lower than the optimal emissions level of a firm generating externalities with long lags. Thus, if a regulator seeks to control multiple firms that generate externalities with different lag times, the ranking of these firms in terms of their optimal emissions rates may vary depending on the value of key environmental and socioeconomic parameters.

To make our theoretical results more concrete, we present some visual examples from simulations based on the most complex scenario analyzed in the previous section, which involved an exponential damage function and uncertainty type 2 (see Sect. 4.3). Figure 2 presents the optimal emissions levels for Firms A and B under different values of the variance parameter σ_θ^2. The figure illustrates how for low levels of uncertainty, the optimal emissions level for Firm B is higher than the optimal emissions level for Firm A. This is due to the fact that for these low values of σ_θ^2, the discounting effect dominates the "precautionary" effect, thus favoring emissions from the firm associated with longer time lags (Firm B). However, at high levels of uncertainty, the "precautionary" effect dominates the discounting effect, making it optimal to allow greater emissions from the firm associated with shorter lags (Firm A).

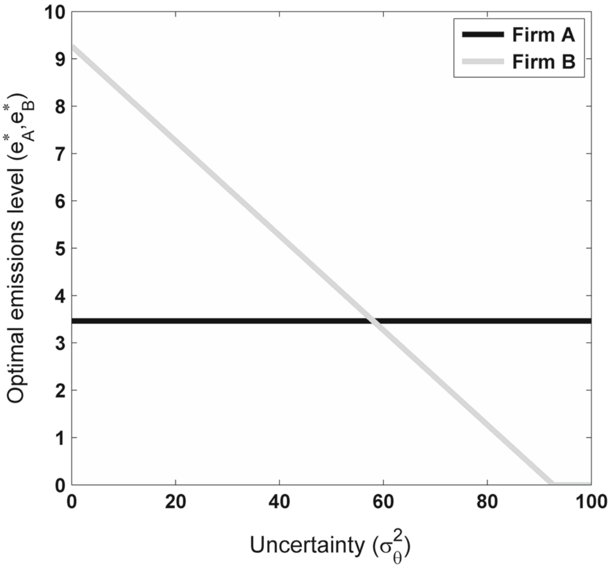

Fig. 2 Optimal emissions levels for Firms A and B at different values of the variance parameter σ_θ^2

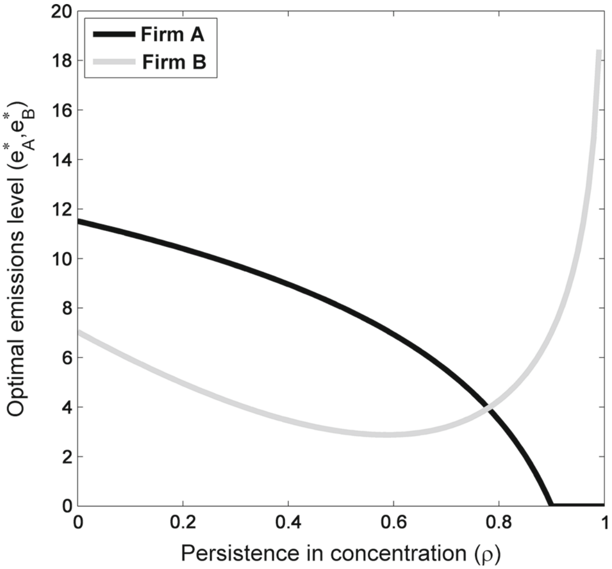

Fig. 3 Optimal emissions levels for Firms A and B at different values of the persistence parameter ρ

Figure 3 illustrates the optimal emissions levels for the two firms under different values of the persistence parameter ρ, holding other parameters constant. This simulation illustrates the complexity that can arise in the optimal allocation of emissions when lags and uncertainty are present simultaneously. As in Fig. 2, the ranking of the two firms with respect to their optimal emissions switches at a given point within the range of the parameter of interest. However, Fig. 3 illustrates an added nonlinearity in the relationship between optimal emissions by the two firms and the persistence parameter ρ. In fact, the optimal emissions level for Firm B is non-monotonic in the level of persistence; at low levels of ρ, increases in persistence require the regulator to decrease optimal emissions by Firm B, but at high levels of persistence, increases in persistence imply increases in optimal emissions by Firm B.

Admittedly, the choice of parameter values in these simulations is arbitrary and is not associated with an actual environmental management problem. We also acknowledge that the two-period, two-firm case analyzed in Sect. 3 is a simplification; all of the components of the social planner's problem, including the structure of firm abatement costs, the environmental damages, and stochastic process are likely to be much more complicated in reality. However, our results show that the implications of time lags and uncertainty can introduce significant complexity in the optimal environmental policy, even with a simple depiction of the underlying physical and economic systems. As a result, optimal policies for real-world environmental problems involving lagged and stochastic externalities are also likely to be complex, and regulators may benefit from an awareness of the potential impact of lags and uncertainty and use appropriately calibrated numerical models to formulate an actual policy.

There are several ways in which a regulator could implement the type of optimal policies derived in this analysis. Other than a straightforward command-and-control type policy in which the regulator caps each polluter's emissions at the optimal level, the optimal allocation can also be induced using a market-based approach such as a Pigouvian tax or tradable emissions permit market. Under such a setup, polluters that seek to minimize their private abatement costs will choose the socially optimal level of abatement if the pricing mechanism reflects the optimality conditions associated with each social planner's problem described in Sect. 4. For example, in the case of the exponential damage function and uncertainty type 1 (Sect. 4.2), if we let p_A and p_B equal the emissions permit prices charged to Firms A and B for the right to emit a unit of pollution in the first time period, then the optimality conditions imply that the optimal permit pricing schedule is:

$$p_A = \psi \exp\left(\psi\left(e_A + \theta_1\right)\right), \tag{18}$$

$$p_B = \psi\beta \exp\left(\psi\left(e_B + \mu\theta_1\right) + \frac{1}{2}\psi^2\sigma_\epsilon^2\right). \tag{19}$$

Accordingly, if the two firms are allowed to trade emissions permits, they will be subject to a trading ratio; if Firm A wishes to purchase a permit from Firm B, the trade must occur at a ratio equal to:

$$\tau = \beta \exp\left(\psi\left(e_B - e_A + \theta_1(\mu - 1) + \frac{1}{2}\psi\sigma_\epsilon^2\right)\right), \tag{20}$$

which is the ratio of expected marginal damages associated with emissions by the two firms.

It is important to note that the regulator does not need to know the polluters' abatement cost functions in order to implement this market-based solution. However, the regulator does need to know the lag structure associated with each polluter's emissions, the environmental damage function, and the associated stochastic process in order to correctly characterize the marginal damages of emissions over time in the pricing mechanism. In addition, the design of such policy instruments is more complicated than typical Pigouvian taxes or permit markets addressed by environmental economists due to the fact that polluters must be regulated differentially; polluters that cause externalities with different time lags must face different prices for emissions. However, such differentiated policies have been discussed extensively in the literature for the regulation of air pollutants (Atkinson and Lewis 1974; Krupnick 1986) and for the management of water resources (Farrow et al. 2005; Kuwayama and Brozović

2013). In fact, differentiated market-based regulations already exist, for example, in the form of water pollution trading programs (Fisher-Vanden and Olmstead 2013).

A potential direction for future work is to explore the implications of time lags on the regulator's choice between price-based policies (e.g. taxes) and quantity-based policies (e.g. tradable permits), based on the seminal work of Weitzman (1974). For example, Newell and Pizer (2003) compared the performance of these two types of policies for controlling stock externalities, with a focus on the roles of correlation of cost shocks across time, discounting, and stock decay. In an application to the problem of greenhouse gases and climate change, the authors find that a price-based instrument generates several times the expected net benefits of a quantity instrument. An interesting extension would be to assess whether the presence of time lags in climate change impacts alters this result significantly.

Our analysis also suggests that quadratic functional forms for damage functions, which are widely used to quantify the environmental costs of pollution and overexploitation of resources, may not be appropriate for deriving optimal polices to manage externalities with heterogeneous lags. Quadratic functions are associated with marginal damages that are linear in pollution and will thus not reflect the societal cost of increasing uncertainty. We have shown that the third derivative of the damage function can play an important role in forming an optimal pollution policy when (a) there is a tradeoff between two externalities of different lag length and (b) there future state of the affected resource is uncertain. Furthermore, in many discussions regarding economic activities that generate environmental damages in the future, the effects of time lags and persistence are often conflated, when in fact they are distinct dynamic elements. Time lags in externalities can exist without persistence, and vice versa. Our modeling in Sect. 3 treated these two elements separately, and our results show that they have qualitatively different implications on optimal environmental management.

Finally, our analysis revealed the importance of initial conditions when regulators seek to manage a lagged and stochastic externality. Specifically, in all four of the scenarios developed in Sect. 3, emissions levels were in part determined by shocks observed during the time period in which the regulator sets the optimal allocation. The implication of this result in the context of water resource management, for example, is that the optimal water allocation across multiple users will be different depending on whether the allocation decision is made in a wet year or a dry year. The fact that observed stochastic shocks matter in determining an optimal policy may be an argument in favor of increased monitoring and evaluation of current environmental conditions.

6 Conclusion

There are many processes in nature that exhibit time lags and uncertainty, but the joint impact of these two features on the optimal management of environmental externalities has thus far not been explored in the literature. In this paper, we developed a theoretical model that demonstrates the potential for lags and uncertainty to alter the optimal allocation of emissions across different firms. Our model with two polluters and two time periods, while simple, provided clarity and allowed us to focus on the basic economic principles that arise from this problem. We have shown that such optimal allocations will require differentiated regulation of polluters, and that it is not obvious whether greater control should be exerted on polluters that generate externalities with longer lags or on polluters that generate externalities with shorter lags. Our model with two polluters and two time periods revealed that the optimal ranking of polluters with respect to the length of the time lag associated with their externality will depend on (a) the discount rate, (b) conditional expectations of future states of the polluted resource, (c) persistence of the pollutant, and (d) initial conditions. The relationship between these factors and optimal emissions depends crucially on the form of the environmental damage function, specifically, on whether marginal damages are linear or not in the level of ambient contaminant concentration.

A limitation of our study is the relative simplicity of the two-polluter, two-period model, which was necessary to obtain analytical results that provide an intuitive understanding of how environmental policies can be designed to manage externalities with lags and uncertainty. Important simplifying assumptions included a linear abatement cost structure that was common across polluters and the use of quadratic or exponential forms for environmental damage functions. Furthermore, our optimal pollution problem was modeled as a one-time decision, whereas real-world pollution problems are likely to involve repeated decisions over time regarding emissions.

A multi-period version of our model would add interesting and complex features to an optimal emissions policy. For example, under uncertainty type 2, if the social planner was required to choose Firm A's emissions in time period 2 as well as in time period 1, she may choose to reduce emissions in time period 1 not only in response to an unfavorable initial shock (the effects of which will persist into the period 2) but also in order to mitigate future risk and allow for more emissions in time period 2. A dynamic version of our model would also require consideration of "open loop" versus "closed loop" optimal solutions and the question of whether the social planner is able to commit to a given emissions path after the first time period.

Future research can also model problems involving environmental externalities with time lags and uncertainty when regulators do not act as social planners. Lieb (2004) develops an overlapping generations model with myopic governments that regulate a flow pollutant that causes immediate damages and a stock pollutant that harms the environment only in the future. The author shows that it is possible for emissions of the flow pollutant to follow an environmental Kuznets curve as nations curtail emissions of the flow pollutant to counteract the negative impacts of the stock pollutant, the emissions of which increase monotonically as incomes in the nation rise. An extension of this model could assess whether uncertainty in the evolution of the stock pollutant reinforces or diminishes the likelihood of observing an environmental Kuznets curve.

Designing optimal emissions policies in real-world settings will require numerical modeling that can account for more of the dynamic complexities inherent to natural and human systems. The results from this paper can provide some intuition behind the results of future studies that implement such numerical models of lagged and stochastic externalities that are more realistic but too complex to explore analytically.

Acknowledgments The authors wish to thank Amy Ando, John Braden, Dallas Burtraw, Ximing Cai, Sahan Dissanayake, Dave Finnoff, Iddo Kan, Alan Krupnick, Chuck Mason, Ariel Ortiz-Bobea, Juan Robalino, Jay Shogren, Fred Sterbenz, Tom Tietenberg, Al Valocchi, Klaas van 't Veld, Guillermo Vuletin, Quinn Weninger, and three anonymous referees for valuable comments and suggestions. This project was completed with financial support from the National Science Foundation, EAR 0709735.

Appendix: Mathematical Derivations

Optimal Emission Levels Under Uncertainty Type 1 and Quadratic Damages

Given the form of the abatement cost function in (1), the quadratic form of the damage function in (2), and the stochastic process described in Sect. 3.3.1, the social planner's objective is to:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\alpha(e_A+\theta_1)+\frac{\gamma}{2}(e_A+\theta_1)^2\right]$$
$$+\mathbb{E}\left\{\beta\left[\alpha(e_B+\theta_2)+\frac{\gamma}{2}(e_B+\theta_2)^2\right]\Big|\,\theta_1\right\}. \tag{21}$$

We can make use of the fact that if a random variable y has mean $\mathbb{E}[y]$ and variance σ_y^2, then $\mathbb{E}(\alpha y+\frac{\gamma}{2}y^2)=\alpha\mathbb{E}[y]+\frac{\gamma}{2}[\sigma_y^2+(\mathbb{E}[y])^2]$. Since $\mathbb{E}[\theta_2|\theta_1]=\mu\theta_1$ and $\mathrm{Var}[\theta_2|\theta_1]=\sigma_\epsilon^2$ by definition, the social planner's objective can be rewritten as:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\alpha(e_A+\theta_1)+\frac{\gamma}{2}(e_A+\theta_1)^2\right]$$
$$+\beta\left\{\alpha(e_B+\mu\theta_1)+\frac{\gamma}{2}\left[\sigma_\epsilon^2+(e_B+\mu\theta_1)^2\right]\right\}. \tag{22}$$

The first-order conditions for an interior solution to this minimization problem are:

$$-\phi+\gamma\left(e_A^*+\theta_1\right)+\alpha=0, \tag{23}$$
$$-\phi+\beta\left[\gamma(e_B^*+\mu\theta_1)+\alpha\right]=0. \tag{24}$$

Equation (23) can be rewritten to obtain the closed-form solution for Firm A's optimal emissions:

$$e_A^*=\frac{\phi-\alpha}{\gamma}-\theta_1. \tag{25}$$

Equation (24) can be rearranged to obtain the closed-form solution for Firm B's optimal emissions:

$$e_B^*=\frac{1}{\gamma}\left(\frac{\phi}{\beta}-\alpha\right)-\mu\theta_1. \tag{26}$$

Optimal Emission Levels Under Uncertainty Type 1 and Exponential Damages

Given the abatement cost function in (1), the exponential form of the damage function in (3), and the stochastic process described in Sect. 3.3.1, the social planner's objective is to:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\exp(\psi(e_A+\theta_1))-1\right]$$
$$+\mathbb{E}\left\{\beta\left[\exp(\psi(e_B+\theta_2))-1\right]\big|\,\theta_1\right\}. \tag{27}$$

We can make use of the fact that if a random variable y is normally distributed with mean $\mathbb{E}[y]$ and variance σ_y^2, then $\mathbb{E}[\exp(y)]=\exp(\mathbb{E}[y]+\frac{\sigma_y^2}{2})$. Since $\mathbb{E}[\theta_2|\theta_1]=\mu\theta_1$ and $\mathrm{Var}[\theta_2|\theta_1]=\sigma_\epsilon^2$ by definition, the social planner's objective can be rewritten as:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\exp(\psi(e_A+\theta_1))-1\right]$$
$$+\beta\left[\exp\left(\psi\left(e_B+\mu\theta_1\right)+\frac{\psi\sigma_\epsilon^2}{2}\right)-1\right]. \tag{28}$$

The first-order conditions for an interior solution to this minimization problem are:

$$-\phi+\psi\exp\left(\psi\left(e_A^*+\theta_1\right)\right)=0, \tag{29}$$
$$-\phi+\psi\beta\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)=0. \tag{30}$$

Taking the natural logarithm of both sides of Eqs. (29) and (30) rearrangement yields the closed-form solution for Firms A and B optimal emissions:

$$e_A^* = \frac{1}{\psi} \ln\left(\frac{\phi}{\psi}\right) - \theta_1, \tag{31}$$

$$e_B^* = \frac{1}{\psi} \ln\left(\frac{\phi}{\psi\beta}\right) - \mu\theta_1 - \frac{\psi\sigma_\epsilon^2}{2}. \tag{32}$$

Optimal Emission Levels Under Uncertainty Type 2 and Quadratic Damages

Given the form of the abatement cost function in (1) and the quadratic form of the damage function in (2), the social planner's objective is to:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\alpha(\rho x_0+e_A+\theta_1)+\frac{\gamma}{2}(\rho x_0+e_A+\theta_1)^2\right]$$
$$+\mathbb{E}\left\{\beta\left[\alpha\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B+\theta_2\right)+\frac{\gamma}{2}\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B+\theta_2\right)^2\middle|\,\theta_1\right]\right\}. \tag{33}$$

We can make use of the fact that if a random variable y has mean $\mathbb{E}[y]$ and variance σ_y^2, then $\mathbb{E}(\alpha y+\frac{\gamma}{2}y^2)=\alpha\mathbb{E}[y]+\frac{\gamma}{2}[\sigma_y^2+(\mathbb{E}[y])^2]$. Since $\mathbb{E}[\theta_t]=0$ and $\text{Var}[\theta_t]=\sigma_\theta^2$ by definition, the objective function can be rewritten as:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\alpha(\rho x_0+e_A+\theta_1)+\frac{\gamma}{2}(\rho x_0+e_A+\theta_1)^2\right]$$
$$+\beta\left\{\alpha\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B\right)+\frac{\gamma}{2}\left[\sigma_\theta^2+\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B\right)^2\right]\right\}. \tag{34}$$

The first-order conditions for an interior solution to this minimization problem are:

$$-\phi+\alpha+\gamma\left(\rho x_0+e_A^*+\theta_1\right)+\beta\left[\alpha\rho+\gamma\left(\rho^2x_0+\rho e_A^*+\rho\theta_1+e_B^*\right)\rho\right]=0, \tag{35}$$

$$-\phi+\beta\left[\alpha+\gamma\left(\rho^2x_0+\rho e_A^*+\rho\theta_1+e_B^*\right)\right]=0. \tag{36}$$

Solving this system of two equations and two unknowns yields closed-form solutions for optimal emissions by Firms A and B:

$$e_A^* = \frac{\phi-\phi\rho-\alpha}{\gamma}-\rho x_0-\theta_1, \tag{37}$$

$$e_B^* = \frac{\phi-\alpha\beta-\beta\rho(\phi-\phi\rho-\alpha)}{\gamma\beta}. \tag{38}$$

Optimal Emission Levels Under Uncertainty Type 2 and Exponential Damages

Given the form of the abatement cost function in (1) and the form of the damage function in (3), the social planner's objective is to:

$$\min_{e_A,e_B} \quad \phi(\bar{e}-e_A)+\phi(\bar{e}-e_B)+\left[\exp\left(\psi\left(\rho x_0+e_A+\theta_1\right)\right)-1\right]$$
$$+\mathbb{E}\left\{\beta\left[\exp\left(\psi\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B+\theta_2\right)\right)-1\right]\middle|\,\theta_1\right\}. \tag{39}$$

We can make use of the fact that if a random variable y is normally distributed with mean $\mathbb{E}[y]$ and variance σ_y^2, then $\mathbb{E}[\exp(y)]=\exp(\mathbb{E}[y]+\frac{\sigma_y^2}{2})$. Since $\mathbb{E}[\theta_t]=0$ and $\text{Var}[\theta_t]=\sigma_\theta^2$ by definition, the objective function can be rewritten as:

$$\min_{e_A, e_B} \quad \phi(\bar{e} - e_A) + \phi(\bar{e} - e_B) + \left[\exp\left(\psi\left(\rho x_0 + e_A + \theta_1\right)\right) - 1\right]$$
$$+ \beta \left[\exp\left(\psi(\rho^2 x_0 + \rho e_A + \rho\theta_1 + e_B) + \frac{\psi^2\sigma_\theta^2}{2}\right) - 1\right]. \tag{40}$$

The first-order conditions for an interior solution to this minimization problem are:

$$-\phi + \psi \exp\left(\psi\left(\rho x_0 + e_A^* + \theta_1\right)\right)$$
$$+ \beta\psi\rho \exp\left(\psi\left(\rho^2 x_0 + \rho e_A^* + \rho\theta_1 + e_B^*\right) + \frac{\psi^2\sigma_\theta^2}{2}\right) = 0, \tag{41}$$

$$-\phi + \beta\psi \exp\left(\psi\left(\rho^2 x_0 + \rho e_A^* + \rho\theta_1 + e_B^*\right) + \frac{\psi^2\sigma_\theta^2}{2}\right) = 0. \tag{42}$$

Solving this system of two equations and two unknowns yields closed-form solutions for optimal emissions by Firms A and B:

$$e_A^* = \frac{1}{\psi} \ln\left(\frac{\phi - \phi\rho}{\psi}\right) - \rho x_0 - \theta_1, \tag{43}$$

$$e_B^* = \frac{1}{\psi}\left[\ln\left(\frac{\phi}{\beta\psi}\right) - \rho \ln\left(\frac{\phi - \phi\rho}{\psi}\right)\right] - \frac{\psi\sigma_\theta^2}{2}. \tag{44}$$

Optimal Emission Levels Under Generic Abatement Costs

In this appendix, we derive relationships between key model parameters and optimal emissions levels using generic abatement cost functions and an exponential damage function.

Generic Abatement Costs, Exponential Damages, and Uncertainty Type 1

Under uncertainty type 1, the social planner's objective is to:

$$\min_{e_A, e_B} \quad C(\bar{e} - e_A) + C(\bar{e} - e_B) + \exp(\psi(e_A + \theta_1)) - 1$$
$$+ \mathbb{E}\left\{\beta\left[\exp(\psi(e_B + \theta_2)) - 1\right] | \theta_1\right\}. \tag{45}$$

As in "Optimal Emission Levels Under Uncertainty Type 1 and Exponential Damages" section in "Appendix", we can rewrite the social planner's objective as:

$$\min_{e_A, e_B} \quad C(\bar{e} - e_A) + C(\bar{e} - e_B) + \exp(\psi(e_A + \theta_1)) - 1$$
$$+ \beta\left[\exp\left(\psi(e_B + \mu\theta_1) + \frac{\psi\sigma_\epsilon^2}{2}\right) - 1\right]. \tag{46}$$

The first-order conditions for an interior solution to this minimization problem are:

$$-C'\left(\bar{e} - e_A^*\right) + \psi \exp\left(\psi\left(e_A^* + \theta_1\right)\right) = 0, \tag{47}$$

$$-C'\left(\bar{e} - e_B^*\right) + \beta\psi \exp\left(\psi\left(e_B^* + \mu\theta_1\right) + \frac{\psi^2\sigma_\epsilon^2}{2}\right) = 0. \tag{48}$$

In order to examine how e_A^* and e_B^* vary with β, we take the total derivative of the first-order conditions with respect to β and rearrange to get:

$$C''\left(\bar{e}-e_A^*\right)\frac{de_A^*}{d\beta}+\psi^2\exp\left(\psi\left(e_A^*+\theta_1\right)\right)\frac{de_A^*}{d\beta}=0, \tag{49}$$

$$C''\left(\bar{e}-e_B^*\right)\frac{de_B^*}{d\beta}+\beta\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)\frac{de_B^*}{d\beta}$$
$$+\psi\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)=0. \tag{50}$$

Equations (49) and (50) can be rearranged to obtain expressions for the change in optimal emissions for Firms A and B in response to a change in β:

$$\frac{de_A^*}{d\beta}=0, \tag{51}$$

$$\frac{de_B^*}{d\beta}=-\frac{\psi\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}{C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}. \tag{52}$$

Assuming that $C''(\cdot)>0$, Eq. (52) implies that $\frac{de_B^*}{d\beta}<0$. A similar procedure yields expressions for the change in optimal emissions in response to changes in other modeling parameters, θ_1, μ, and σ_ϵ^2:

$$\frac{de_A^*}{d\theta_1}=-\frac{\psi^2\exp\left(\psi\left(e_A^*+\theta_1\right)\right)}{C''\left(\bar{e}-e_A^*\right)+\psi^2\exp\left(\psi\left(e_A^*+\theta_1\right)\right)}<0, \tag{53}$$

$$\frac{de_B^*}{d\theta_1}=-\frac{\beta\mu\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}{C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}<0, \tag{54}$$

$$\frac{de_A^*}{d\mu}=0, \tag{55}$$

$$\frac{de_B^*}{d\mu}=-\frac{\beta\psi^2\theta_1\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}{C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}\lesseqgtr 0, \tag{56}$$

$$\frac{de_A^*}{d\sigma_\epsilon^2}=0, \tag{57}$$

$$\frac{de_B^*}{d\sigma_\epsilon^2}=-\frac{\beta\psi^3\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)}{2\left[C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp\left(\psi\left(e_B^*+\mu\theta_1\right)+\frac{\psi^2\sigma_\epsilon^2}{2}\right)\right]}<0. \tag{58}$$

Generic Abatement Costs, Exponential Damages, and Uncertainty Type 2

Under uncertainty type 2, the social planner's objective is to:

$$\min_{e_A,e_B}\quad C(\bar{e}-e_A)+C(\bar{e}-e_B)+\exp(\psi(\rho x_0+e_A+\theta_1))-1$$
$$+\mathbb{E}\left\{\beta\left[\exp\left(\psi\left(\rho^2x_0+\rho e_A+\rho\theta_1+e_B+\theta_2\right)\right)\right]-1\middle|\theta_1\right\}. \tag{59}$$

As in "Optimal Emission Levels Under Uncertainty Type 2 and Exponential Damages" section in "Appendix", we can rewrite the social planner's objective as:

$$\min_{e_A,e_B} \quad C(\bar{e}-e_A)+C(\bar{e}-e_B)+\exp(\psi(\rho x_0+e_A+\theta_1))-1$$
$$+\beta\left[\exp\left(\psi\left(\rho^2 x_0+\rho e_A+\rho\theta_1+e_B\right)+\frac{\psi^2\sigma_\theta^2}{2}\right)-1\right]. \quad (60)$$

The first-order conditions for an interior solution to this minimization problem are:

$$-C'\left(\bar{e}-e_A^*\right)+\psi\exp\left(\psi\left(\rho x_0+e_A^*+\theta_1\right)\right)$$
$$+\beta\rho\psi\exp\left(\psi\left(\rho^2 x_0+\rho e_A^*+\rho\theta_1+e_B^*\right)+\frac{\psi^2\sigma_\theta^2}{2}\right)=0, \quad (61)$$
$$-C'\left(\bar{e}-e_B^*\right)+\beta\psi\exp\left(\psi\left(\rho^2 x_0+\rho e_A^*+\rho\theta_1+e_B^*\right)+\frac{\psi^2\sigma_\theta^2}{2}\right)=0. \quad (62)$$

In order to examine how e_A^* and e_B^* vary with β, we take the total derivative of the first-order conditions with respect to β and rearrange to get:

$$\left[C''\left(\bar{e}-e_A^*\right)+\psi^2\exp(\mathbf{\Gamma}_1)+\beta\rho^2\psi^2\exp(\mathbf{\Gamma}_2)\right]\frac{de_A^*}{d\beta}+\beta\rho\psi^2\exp(\mathbf{\Gamma}_2)\frac{de_B^*}{d\beta}$$
$$=-\psi\rho\exp(\mathbf{\Gamma}_2), \quad (63)$$
$$\beta\rho\psi^2\exp(\mathbf{\Gamma}_2)\frac{de_A^*}{d\beta}+\left[C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp(\mathbf{\Gamma}_2)\right]\frac{de_B^*}{d\beta}=-\psi\exp(\mathbf{\Gamma}_2), \quad (64)$$

where

$$\mathbf{\Gamma}_1=\psi\left(\rho x_0+e_A^*+\theta_1\right), \quad (65)$$
$$\mathbf{\Gamma}_2=\psi\left(\rho^2 x_0+\rho e_A^*+\rho\theta_1+e_B^*\right)+\frac{\psi^2\sigma_\theta^2}{2}. \quad (66)$$

Equations (63) and (64) can be rewritten in matrix form:

$$\begin{bmatrix} C''\left(\bar{e}-e_A^*\right)+\psi^2\exp(\mathbf{\Gamma}_1)+\beta\rho^2\psi^2\exp(\mathbf{\Gamma}_2) & \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) \\ \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) & C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp(\mathbf{\Gamma}_2) \end{bmatrix}$$
$$\times\begin{bmatrix} \frac{de_A^*}{d\beta} \\ \frac{de_B^*}{d\beta} \end{bmatrix}=\begin{bmatrix} -\psi\rho\exp(\mathbf{\Gamma}_2) \\ -\psi\exp(\mathbf{\Gamma}_2) \end{bmatrix}.$$

We define the following three matrices:

$$\mathbf{G}=\begin{bmatrix} C''\left(\bar{e}-e_A^*\right)+\psi^2\exp(\mathbf{\Gamma}_1)+\beta\rho^2\psi^2\exp(\mathbf{\Gamma}_2) & \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) \\ \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) & C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp(\mathbf{\Gamma}_2) \end{bmatrix},$$
$$\mathbf{G_1}=\begin{bmatrix} -\psi\rho\exp(\mathbf{\Gamma}_2) & \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) \\ -\psi\exp(\mathbf{\Gamma}_2) & C''\left(\bar{e}-e_B^*\right)+\beta\psi^2\exp(\mathbf{\Gamma}_2) \end{bmatrix},$$
$$\mathbf{G_2}=\begin{bmatrix} C''\left(\bar{e}-e_A^*\right)+\psi^2\exp(\mathbf{\Gamma}_1)+\beta\rho^2\psi^2\exp(\mathbf{\Gamma}_2) & -\psi\rho\exp(\mathbf{\Gamma}_2) \\ \beta\rho\psi^2\exp(\mathbf{\Gamma}_2) & -\psi\exp(\mathbf{\Gamma}_2) \end{bmatrix}.$$

Assuming that $C''(\cdot) > 0$, we can determine whether the determinants of the above three matrices are positive or negative:

$$\begin{aligned}\det(\mathbf{G}) = {} & C''\left(\bar{e} - e_A^*\right) C''\left(\bar{e} - e_B^*\right) + \psi^2 C''\left(\bar{e} - e_B^*\right) \exp(\boldsymbol{\Gamma}_1) \\ & + \beta\rho^2\phi^2 C''\left(\bar{e} - e_B^*\right) \exp(\boldsymbol{\Gamma}_2) + \beta\psi^2 C''\left(\bar{e} - e_A^*\right) \exp(\boldsymbol{\Gamma}_2) \\ & + \beta\psi^4 \exp(\boldsymbol{\Gamma}_1) \exp(\boldsymbol{\Gamma}_2) > 0, \end{aligned} \quad (67)$$

$$\det(\mathbf{G_1}) = -\rho\psi C''\left(\bar{e} - e_B^*\right) \exp(\boldsymbol{\Gamma}_2) < 0, \quad (68)$$

$$\det(\mathbf{G_2}) = -\psi \exp(\boldsymbol{\Gamma}_2)[C''\left(\bar{e} - e_A^*\right) + \psi^2 \exp(\boldsymbol{\Gamma}_1)] < 0. \quad (69)$$

By Cramer's rule, it follows that:

$$\frac{de_A^*}{d\beta} = \frac{\det(\mathbf{G_1})}{\det(\mathbf{G})} < 0, \quad (70)$$

$$\frac{de_B^*}{d\beta} = \frac{\det(\mathbf{G_2})}{\det(\mathbf{G})} < 0, \quad (71)$$

A similar procedure yields expressions for the change in optimal emissions in response to changes in other modeling parameters, x_0, θ_1, ρ, and σ_θ^2:

$$\frac{de_A^*}{dx_0} < 0, \ \frac{de_B^*}{dx_0} < 0, \ \frac{de_A^*}{d\theta_1} < 0, \ \frac{de_B^*}{d\theta_1} < 0, \ \frac{de_A^*}{d\rho} \lesseqgtr 0, \ \frac{de_B^*}{d\rho} \lesseqgtr 0, \ \frac{de_A^*}{d\sigma_\theta^2} < 0, \ \frac{de_B^*}{d\sigma_\theta^2} < 0. \quad (72)$$

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