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Which volatility model for option valuation in China? Empirical evidence from SSE 50 ETF options

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ABSTRACT
In early 2015, China launched its first exchange-traded option, the Shanghai Stock Exchange (SSE) 50 ETF option, to meet the increasing demand for financial derivatives. In this article, we provide an intensive empirical investigation of popular discrete-time volatility models in terms of their pricing performance when applied to SSE 50 ETF options. We find that the newly developed models with realized measures significantly outperform conventional GARCH-type models based on daily returns only. In contrast with the U.S. market, our empirical results suggest that the leverage effect is very weak in the Chinese option market.

KEYWORDS
Discrete-time models; realized measures; option pricing; Chinese market

JEL CLASSIFICATION
C10; C22; C80

I. Introduction

Options are one of the most important types of fundamental derivatives in the global market. They are widely used in areas such as risk management and the formulation of structured products. However, exchange-based options were not available in the Chinese market until nearly 25 years after the establishment of the Shanghai and Shenzhen stock exchanges, namely when the Shanghai Stock Exchange (SSE) 50 ETF Option was launched in February 2015. As the only domestically traded exchange-based option in the market, the trading volume and open interest of the 50 ETF option have increased nearly 70-fold over the past 3 years and the number of qualified investors has increased almost 100-fold[1]

There are several reasons why it is important to examine the 50 ETF option more closely. First, the option covers a great amount of market capitalization. The underlying asset of the option is a highly liquid ETF that covers the 50 largest blue-chip stocks traded on the SSE, which constitute 25% of the SSE's market capitalization. The total capitalization of the Chinese stock market ranks third[2] worldwide and first among emerging markets. Second, the option market in China is different from its developed market counterparts in many ways. The option is a physically delivered ETF option rather than a cash-settled index option. The contract is adjusted with dividends from the underlying asset, which is uncommon in standard option contracts. The number of available strikes is quite limited and the cost of short selling is much higher than in the U.S. market. Third, the commonly known stylized facts may not be applicable to the market. The implied volatility is much higher than in the U.S. market and the implied smirk is weak or skewed to the right rather than the left (Yue et al. 2019). The fact that 30 component stocks of the ETF are also included in the MSCI China A Onshore Index suggests that our discussion may also have global interest. Among many potential areas of interest, this article focuses on the pricing performance among discrete-time volatility models for the 50 ETF option, as it is possible that the outlined differences have implications for the model structures and data.

Research on option-pricing models relies heavily on models to model volatility dynamics. Aside from continuous-time models (such as Black and Scholes 1973; Heston 1993), discrete-time volatility models have also been considered as a platform for

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[1]The number of qualified investors on the first trading day in 2015 was 2626. This number increased to 260,040 by the early 2018..

[2]China's market surpassed Japan in market value in November 2014 to become the world's second-largest and kept that rank until late August 2018. Despite the 2018 crash, China's equity market remains among the top three equity markets and is the largest emerging market worldwide.

option-pricing practice. Motivated by the success of GARCH models in financial econometrics, Duan (1995) pioneered using GARCH models[3] as a pricing platform for options, and Heston and Nandi (2000) provided a structure that made a closed-form pricing formula for European options available. Christoffersen and Jacobs (2004) highlighted the advantages of GARCH option-pricing models in terms of estimation and time efficiency for pricing large-scale option panels[4] These studies have typically focused on the U.S. option market and have found that better results can be obtained from models which include the leverage effect and joint estimation of both the underlying and the option panels. Based on the development of model-free estimations of daily variance (Andersen and Bollerslev 1998), a list of new models was proposed (Andersen et al. 2003; Corsi 2009; Hansen, Huang, and Shek 2012; Hansen and Huang 2016). Some of these models have been tailored for option-pricing purposes; they include the GARV model (Christoffersen et al. 2014), the Realized GARCH model (Huang, Wang, and Hansen 2017), and the HAR-type models (Majewski, Bormetti, and Corsi 2015). For all of these models, researches have highlighted the importance of realized measures in improving the pricing performance of discrete-time models.

There are several papers comparing the pricing performance across different models. Bakshi, Cao, and Chen (1997) compared a series of continuous-time volatility models and highlighted important model features such as the stochastic volatility and the jump components. Christoffersen and Jacobs (2004) compared a series of GARCH models using the S&P500 options and argued that features such as volatility clustering and the leverage effect are crucial for a better pricing performance. Similarly, Hsieh and Ritchken (2005) compared the affine Heston-Nandi GARCH model (Heston and Nandi 2000) with the non-affine NGARCH model (Engle and Ng 1993) and found better pricing results for the non-affine model. Other researchers have compared models beyond the pricing of European options (e.g. Stentoft 2011), although most research on model comparison has focused on the S&P500 index option.

In contrast to the extensive literature on the U.S. option market, studies of the Chinese option market are limited. Early studies such as Xiong and Yu (2011) have focused on the warrant market bubble from the late 2000. Other studies such as Wang et al. (2017a) and Huang et al. (2018) have discussed the implied volatility and related premium of the 50 ETF options, while Li et al. (2018) have investigated the momentum effect on the option and the underlying market. In terms of pricing models, Yang (2018) proposed a GARCH option-pricing model using daily returns with the double exponential jump (Kou and Wang 2004) feature. To the best of our knowledge, there is little available extensive discussion and comparison of volatility models for the pricing 50 ETF options.

In this article, we provide an extensive comparison of both affine and non-affine discrete-time volatility models through the lens of pricing 50 ETF options. Other than traditional GARCH models, we also include models with newly developed high-frequency data-based volatility measures to provide academics and practitioners with valuable guidelines for choosing pricing platforms for the Chinese option market.

The results show that discrete-time volatility models can be applied to price SSE 50 ETF options with reasonable accuracy. In line with previous studies, we find that non-affine models generally perform better than affine models across the volatility surface. We also find that realized measure-based models have a better in-sample and out-of-sample pricing performance across every sub-sample. As an extension to the current literature on pricing options, we have also compared the performance across different realized measures. Although they are more preferable for modelling and forecasting volatility, complicated-realized measures do not provide better pricing results than the simple-realized variance. A notable difference in the leverage effect is found with the Chinese dataset. While the presence of the leverage effect in the U.S. option prices has been extensively confirmed (Chernov and Ghysels 2000; Heston and Nandi

[3]The main contribution of this article was providing a transformation called the locally risk-neutral valuation relationship (LRNVR), which linked the physical and risk-neutral parameters.

[4]Especially for those with a closed-form pricing formula..

2000; Eraker 2004; Christoffersen and Jacobs 2004; Majewski, Bormetti, and Corsi 2015), this effect is much weaker in the Chinese option market. We also find that the first year of options trading in the Chinese market (i.e. 2015), during which the market suffered from both extreme volatility and a severely limited arbitrage, is distinct from subsequent years; this suggests that the first year should either be dropped from the sample or it should be treated separately when testing pricing models or possible trading strategies.

The remainder of this article is organized as follows. In Section 2, we provide an overview of the SSE 50 ETF option market. In Section 3, we list the models used in the comparison. In Section 4, we briefly introduce the estimation method used. In Section 5, we present and discuss our empirical results. The last section concludes the article.

II. The features of the SSE 50 ETF option market

The underlying asset of the SSE 50 ETF option is the 50 ETF from the Hua Xia Fund Asset Management Company. Covering the 50 largest high-liquidity blue-chip stocks listed in the SSE, this ETF represented around 12% of the total market value (i.e. over 380 billion RMB) of all 170 equity ETFs in China by the end of 2018.

The 50 ETF option was introduced to investors on 9 February 2015, as a European-style physical delivery option written on the 50 ETF. The daily trading volume and open interest have increased dramatically from 2015 to 2018, as seen in Figure 1(c). Compared with the U.S. counterparts, the 50 ETF options have several notable differences in contract specifications, regulations and market conditions.

First, the number of strikes is limited. The 50 ETF option initially had five strikes (1 ATM, 2 OTM and 2 ITM), which increased to nine strikes (1 ATM, 4 OTM, 4 ITM) by the beginning of 2018. Although some new strikes will be added when the dividend adjustment is performed, there have only been three dividend payments so far since the launch of the 50 ETF option.

Second, the option contract will be adjusted when dividends are paid on the underlying ETF. Suppose that the ex-dividend price of the ETF is S and the cash dividend is d, the adjustment will provide option holders with $S/(S-d)$ new contracts at strike $(S-d)K/S$ for one old contract at strike K. Thus, we obtain the following equation:

$$\frac{S}{(S-d)}C_t\left(S-d,\frac{(S-d)}{S}K,T\right)=C_t(S,K,T)$$

This adjustment essentially protects option holders from dividend-induced price fluctuations. This is not a standard feature of index options such as the SPX options.

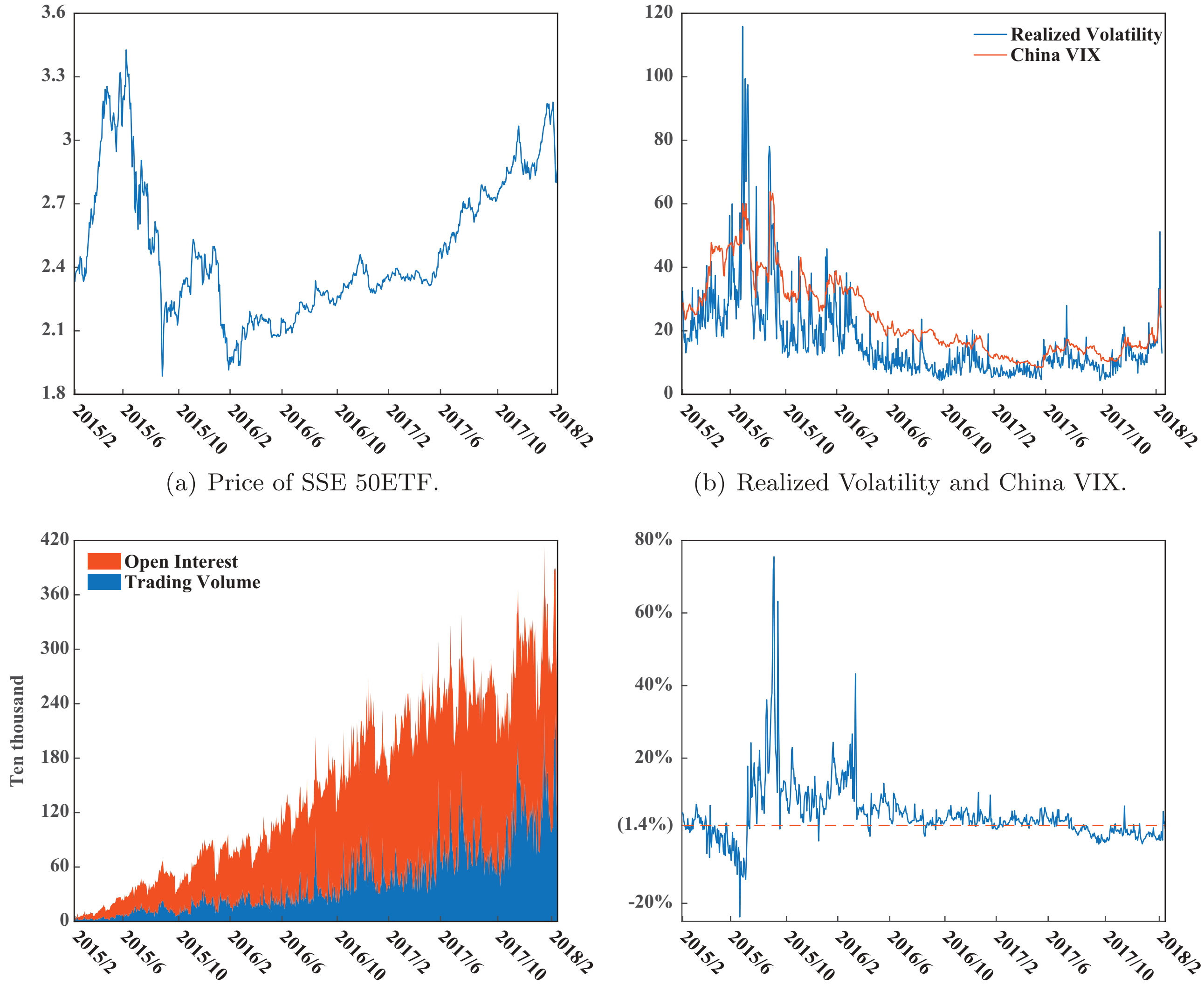
Third, the Chinese market has higher price fluctuations. Figure 1(b) shows the realized volatility calculated from 5 min return of the 50 ETF; it also depicts the China VIX calculated from the 50 ETF options. The average realized volatility from February 2015 to February 2018 for the ETF is around 17.74% (compared with 9.44% for the SPX), and the average VIX for the option is 24.13% (compared with 14.95% for the U.S. VIX). The volatility premium required by 50 ETF traders, defined as VIX – Realized volatility, is 16% higher than that for the SPX options.

Finally, the short-sell cost implied by the option prices is high in 2015 and decreases over time. The implied short-sell cost is calculated following Ofek, Richardson, and Whitelaw (2004) and Bilson, Kang, and Luo (2015), using the put-call parity of European options. If there is no constraint on short selling, then the implied dividend yield of the put-call parity should be close to the actual dividend yield. Specifically, for each option pair i with the same strike and maturity on day t, we derive an implied dividend yield $y_i(t)$ (annualized) using the put-call parity:

$$C_i(t)-P_i(t)=S(t)e^{-y_i(t)T}-Ke^{-rT}$$

Here $C_i(t)$ and $P_i(t)$ are the prices of a call and a put option price pair with the same maturity T and the same strike price K. The variable r represents the risk-free rate. There are N_t option pairs on day t with the same strike and maturity. Figure 1(d) reports the daily average of the implied dividend yield $y(t)=\sum_i y_i(t)/N_t$ and the historical average of the dividend yield. The implied dividend yields are higher than their actual value in most cases, especially in late 2015. This is consistent with the actions taken by regulatory authorities to tighten the constraints on short selling[5] during the market

[5]For example, the China Financial Futures Exchange doubled the margin requirement for short index futures on 8 July 2015.



(a) Price of SSE 50ETF.

(b) Realized Volatility and China VIX.

(c) Daily Activity of SSE 50ETF Option Market.

(d) Put-Call Parity Implied Dividend Yield.

Figure 1. (a) The daily price of SSE 50ETF over the 2015/02 to 2018/02 sample period; (b) corresponding realized volatility (calculated using 5-min returns) and China VIX (extracted from SSE 50ETF option prices and released by the Shanghai stock exchange); (c) trading volume and open interest (daily average). The blue solid line in (d) is the daily average put-call parity implied dividend yield and the red-dashed line is the actual historical average dividend yield during the sample period.

crash. This gap decreases over time, indicating that the option market has become more efficient in recent years.

III. Model comparison

We test nine discrete-time volatility models including the standard GARCH model engle (Engle and Bollerslev 1986); three asymmetric GARCH models, GJR-GARCH (Glosten, Jagannathan, and Runkle 1993), NGARCH (Engle and Ng 1993), and EGARCH (Nelson 1991); two Heston-Nandi GARCH models, HNG (Heston and Nandi 2000) and HNGvd (Christoffersen, Heston, and Jacobs 2013a);[6] and three models with realized measures: Realized GARCH (Hansen and Huang 2016), GARV (Christoffersen et al. 2014), and LHARG (Majewski, Bormetti, and Corsi 2015). Table 1 describes the differences between these models.

[6]HNG and HNGvd are both based on the Heston-Nandi GARCH model. The key difference is the way of risk neutralization. In this article, HNG refers to the model that is risk-neutralized in line with Duan (1995) while HNGvd refers to the model that is risk-neutralized in line with (Christoffersen, Heston, and Jacobs 2013a).

Table 1. Summary of competing models.

	G	GJR	NG	EG	HNG	HNGvd	RGARCH	GARV	LHARG
Linear	✓	✓	✓		✓	✓		✓	✓
Log-linear				✓			✓		
Affine					✓	✓		✓	✓
With RV							✓	✓	✓
With VRP						✓	✓	✓	✓
Closed-form pricing					✓	✓		✓	✓
# of parameters	4	5	5	5	5	6	12	12	12

GARCH models

The GARCH models selected in this study are based on the following GARCH-in-mean framework:

$$r_{t+1} = r + \lambda\sqrt{h_{t+1}} - h_{t+1}/2 + \sqrt{h_{t+1}}z_{t+1}$$

where z_t follows a standard normal distribution. The parameter λ measures the required return of an investor that is proportional to the conditional volatility. The variance equations for the different models are as follows:

- Standard GARCH: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t z_t^2$
- GJR-GARCH: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t z_t^2 + \tau_2 I_{\{z_t<0\}} h_t z_t^2$
- NGARCH: $h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 h_t (z_t - \tau_2)^2$
- EGARCH: $\log h_{t+1} = \beta_0 + \beta_1 \log h_t + \tau_1 z_t + \tau_2(|z_t| - \sqrt{2/\pi})$

The first three models are linear models, whereas the fourth is a log-linear model. Unlike the linear GARCH models, the log-linear GARCH model uses the standardized shock z_t instead of the non-standard shock $\sqrt{h_t}z_t$ to drive the volatility process. It also imposes fewer constraints on the parameters as a way to guaranteeing a positive conditional variance. These advantages come at the cost of a tendency to overreact to volatility shocks and a much more complicated multi-period volatility forecast formula.

Following Duan (1995), the risk-neutral dynamics are linked to their physical counterparts with a locally risk-neutral valuation relationship (LRNVR). Thus, the corresponding risk-neutral dynamics are

$$r_{t+1} = r - h_{t+1}/2 + \sqrt{h_{t+1}}z_{t+1}^*$$

where z_t^* follows a standard normal distribution.

All of these models are non-affine models[7], and the traditional closed-form pricing formula via Fourier inverse transformation is not available. Here, we follow Duan, Gauthier, and Simonato (1999) and price the European call options using an analytical approximation[8]

Heston-Nandi GARCH models

Unlike the GARCH models discussed in the previous section, the Heston-Nandi GARCH (Heston and Nandi 2000) is an affine model with an explicit moment-generation function that can be used to calculate closed-form option prices. This feature makes it a popular benchmark model in discrete-time option pricing. The mean equation for this model is

$$r_{t+1} = r + (\lambda - 1/2)h_{t+1} + \sqrt{h_{t+1}}z_{t+1}$$

where z_t follows a standard normal distribution. The parameter λ measures the required return of an investor that is proportional to the conditional volatility. The variance equation is specified as:

$$h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1 (z_t - \tau_2\sqrt{h_t})^2$$

The risk neutralization of this model can be done in two different ways. The first way is Duan's LRNVR (referred as HNG hereafter) while the second way relies on the variance-dependent pricing kernel proposed by Christoffersen, Heston, and Jacobs (2013a). The latter method explicitly provides an additional parameter in the pricing kernel to accommodate the variance risk premium (referred

[7]In an affine model, the moment-generation function of the return process is an exponential linear function.
[8]The GARCH model can be viewed as a special case ($\tau_2 = 0$) of NGARCH/GJR-GARCH. The approximation formula can be adapted from the one for NGARCH/GJR-GARCH models by using the constraint $\tau_2 = 0$.

as HNGvd hereafter). The corresponding risk-neutral dynamics are:

$$\text{HNG}: r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z^*_{t+1}$$
$$\text{HNGvd}: r_{t+1} = r - \frac{1}{2}h^*_{t+1} + \sqrt{h^*_{t+1}}z^*_{t+1}$$

where z^*_t follows a standard normal distribution. The variance equations are as follows:

$$\text{HNG}: h_{t+1} = \beta_0 + \beta_1 h_t + \tau_1\left(z^*_t - (\tau_2 + \lambda)\sqrt{h_t}\right)^2$$

$$\text{HNGvd}: h^*_{t+1} = \beta^*_0 + \beta_1 h^*_t + \tau^*_1\left(z^*_t - \tau^*_2\sqrt{h^*_t}\right)^2$$

where $h^*_t = \chi h_t$, $\beta^*_0 = \chi\beta_0$, $\tau^*_1 = \chi^2\tau_1$, $\tau^*_2 = 1/2 + (\tau_2 + \lambda - 1/2)/\chi$, and $\chi = 1/(1 + 2\tilde{\chi}\tau_1)$. The parameter $\tilde{\chi}$ represents the free parameter in the variance-dependent pricing kernel.

As the model structures of the HNGvd and the HNG are the same under risk-neutral dynamics, the moment-generation function provided by Heston and Nandi (2000) can be adapted for both models. The European option prices can be calculated using the Fourier inverse transformation.

Models with realized variance

Several models have been proposed for high-frequency data-based volatility modelling. Within the GARCH framework, the Realized GARCH (Hansen, Huang, and Shek 2012; Hansen and Huang 2016), the MEM (Engle and Gallo 2006), and the HEAVY (Shephard and Sheppard 2010) are commonly used complete models that can jointly model returns and realized variance. Reduced-form models such as HAR (Corsi 2009) are also receiving increasing attention. In this study, we focus on three models which have been adapted to the option-pricing practice.

Realized GARCH (RGARCH)

Hansen, Huang, and Shek (2012) propose the Realized GARCH model as an extension of the GARCH-X. Hansen and Huang (2016) introduce the Realized exponential GARCH, which describes the joint dynamics of the returns and the realized variance as follows:

$$r_{t+1} = r + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z_{t+1}$$

$$\log h_{t+1} = \omega + \beta \log h_t + \tau_1 z_t + \tau_2(z^2_t - 1) + \gamma\sigma u_t$$

$$\log x_t = \xi + \phi \log h_t + d_1 z_t + d_2(z^2_t - 1) + \sigma u_t$$

where z_t and u_t are independent standard normal random variables. The volatility-specific shock u_t enables the model to accommodate a variance risk premium in addition to the equity premium. The last measurement equation links u_t with the realized variance and makes the simple ML estimator available.

Following Christoffersen et al. (2014), we use the exponential-affine stochastic discount factor to transform the model into its risk-neutral counterpart:

$$r_{t+1} = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z^*_{t+1}$$

$$\log h_{t+1} = \omega^* + \beta \log h_t + \tau^*_1 z^*_t + \tau_2(z^{*2}_t - 1) + \gamma\sigma u^*_t$$

$$\log x_t = \xi^* + \phi \log h_t + d^*_1 z^*_t + d_2(z^{*2}_t - 1) + \sigma u^*_t$$

where z^*_t and u^*_t are independent standard normal random variables. Moreover, $\omega^* = \omega + \gamma\sigma\chi + \lambda\tau_2(\lambda - 1)$, $\xi^* = \xi + \sigma\chi + \lambda d_2(\lambda - 1)$, $\tau^*_1 = \tau_1 - 2\lambda\tau_2$, and $d^*_1 = d_1 - 2\lambda d_2$. The variable χ represents the parameter associated with the variance risk premium. Huang, Wang, and Hansen (2017) provided an analytical approximation formula by expanding the distribution of the cumulative return based analytical higher moments and normal distributions.

GARV

Another complete model is the Generalized Affine Realized Volatility (GARV) model proposed by Christoffersen et al. (2014). The model decomposes the variance into two parts: the variance calculated via the daily return h^R_t and the variance calculated through the realized variance h^{RV}_t:

$$r_{t+1} = r + \left(\lambda - \frac{1}{2}\right)\bar{h}_{t+1} + \sqrt{\bar{h}_{t+1}}z_{t+1}$$

$$h_{t+1}^R = \omega + \beta h_t^R + \tau_1\left(z_t - \tau_2\sqrt{\bar{h}_t}\right)^2$$
$$h_{t+1}^{RV} = \xi + \phi h_t^{RV} + d_1\left(\epsilon_t - d_2\sqrt{\bar{h}_t}\right)^2$$

$$RV_t = h_t^{RV} + \alpha_2\left[\epsilon_t^2 - 1 - 2d_2\epsilon_t\sqrt{\bar{h}_t}\right]$$
$$\bar{h}_{t+1} = \kappa h_{t+1}^R + (1-\kappa)h_{t+1}^{RV}$$

where (z_t, ϵ_t) follows a standard bivariate normal distribution with the correlation of ρ. The risk-neutral dynamics under the exponential affine stochastic discount factor are:

$$r_{t+1} = r - \frac{1}{2}\bar{h}_{t+1} + \sqrt{\bar{h}_{t+1}}z_{t+1}^*$$

$$h_{t+1}^R = \omega_1 + \beta_1 h_t^R + \tau_1\left(z_t - \tau_2^*\sqrt{\bar{h}_t}\right)^2$$
$$h_{t+1}^{RV} = \xi + \phi h_t^{RV} + d_1\left(\epsilon_t^* - d_2^*\sqrt{\bar{h}_t}\right)^2$$

$$RV_t = h_t^{RV} + \alpha_2\left[\epsilon_t^{*2} - 1 - 2d_2^*\epsilon_t^*\sqrt{\bar{h}_t}\right]$$
$$\bar{h}_{t+1} = \kappa h_{t+1}^R + (1-\kappa)h_{t+1}^{RV}$$

where (z_t^*, ϵ_t^*) also follows a standard bivariate normal distribution with the correlation of ρ. $\tau_2^* = \tau_2 + \lambda$, and $d_2^* = d_2 + \chi$. As in the Realized GARCH model, the parameter χ is associated with the variance risk premium, which is introduced into the model via the discount factor. We report $\gamma = d_1/\alpha_2$ instead of α_2 because the former can be used to measure the contribution of the realized information to the volatility process. Due to the affine structure of the GARV model, a closed-form solution for it is provided by Christoffersen et al. (2014).

LHARG

In addition to GARCH-type models, the availability of high-frequency data and realized measures has boosted the development of reduced-form models such as the HAR model (Corsi 2009). In particular, the HAR model was adapted by using the leverage function of the Heston-Nandi GARCH as well as a gamma distribution (LHARG) in order to price European call options. Majewski, Bormetti, and Corsi (2015) provided a general framework for option pricing with an LHARG model. In this study, we follow the indications of Huang, Tong, and Wang (2019) and use an extended LHARG by adding quarterly and yearly data to more adequately model the long-memory feature of volatility. The dynamics for the physical measures are:

$$r_{t+1} = r + \lambda RV_{t+1} - \frac{1}{2}RV_{t+1} + \sqrt{RV_{t+1}}z_{t+1}$$

$$RV_{t+1}|\mathcal{F}_t \sim \Gamma(\delta, \Theta(\boldsymbol{RV_t}, \boldsymbol{L_t}), \theta)$$

$$\Theta(\boldsymbol{RV_t}, \ell_t) = d + \beta_d RV_t^{(d)} + \beta_w RV_t^{(w)} + \beta_m RV_t^{(m)} + \beta_q RV_t^{(q)} + \beta_y RV_t^{(y)} + \alpha_d \ell_t$$

where z_{t+1} follows i.i.d. standard normal distribution. We define the components as follows:

$$RV_t^{(d)} = RV_t$$
$$RV_t^{(w)} = \left(\sum_{i=1}^{4} RV_{t-i}\right)/4$$
$$RV_t^{(m)} = \left(\sum_{i=5}^{21} RV_{t-i}\right)/17$$

$$RV_t^{(q)} = \left(\sum_{i=22}^{62} RV_{t-i}\right)/41$$
$$RV_t^{(y)} = \left(\sum_{i=63}^{251} RV_{t-i}\right)/189$$

$$l_t = z_t^2 - 1 - 2\gamma z_t\sqrt{RV_t}$$

Here, l_t represents the leverage term which describes the asymmetric reaction of the volatility in response to positive and negative shocks from returns. Following Huang, Tong, and Wang (2019), we only include the daily leverage as a way of keeping the model more concise. The risk-neutral dynamics under the exponential-affine stochastic discount factor are:

$$r_{t+1} = r + \lambda RV_{t+1} - \frac{1}{2}RV_{t+1} + \sqrt{RV_{t+1}}z_{t+1}^*$$

$$RV_{t+1}|\mathcal{F}_t \sim \Gamma(\delta, \Theta^*(\boldsymbol{RV_t}, \ell_t^*), \theta)$$

$$\Theta(\boldsymbol{RV_t}, \ell_t^*) = d^* + \beta_d^* RV_t^{(d)} + \beta_w^* RV_t^{(w)} + \beta_m^* RV_t^{(m)} + \beta_q^* RV_t^{(q)} + \beta_y^* RV_t^{(y)} + \alpha_d^* \ell_t^*$$

The starred risk-neutral parameters are linked to the physical parameters as follows:

$$\beta_d^* = \Delta[\beta_d + \alpha_d(2\gamma\lambda + \lambda^2)] \qquad \alpha_d^* = \Delta\alpha_d$$
$$d^* = \Delta d \qquad \theta^* = \Delta\theta, \qquad \gamma^* = \gamma + \lambda$$

$$\beta_j^* = \Delta\beta_j \ \text{ for } j \in \{w, m, q, y\}$$
$$\ell_t^{(d)} = \epsilon_t^{*2} - 1 - 2\gamma^* \epsilon_t^* \sqrt{RV_t}$$

where $\Delta = \{1 + \theta[(\lambda - 1/2)^2/2 - \chi - 1/8]\}^{-1/2}$. Once again, the parameter χ is associated with the variance risk premium. Majewski, Bormetti, and Corsi (2015) provide the option-pricing formula for arbitrary lags, which can be easily adapted to our setting.

IV. Estimation method

We estimate a model with a joint likelihood for the observed time series and the pricing errors, where the latter are weighted by the Vega[9] Unlike the traditional calibration method focusing only on pricing errors, this approach also takes the model's ability to replicate underlying dynamics into account and received increasing attention in pricing of financial derivatives (Christoffersen et al. 2014; Huang, Wang, and Hansen 2017; Wang et al. 2017b).

Log-likelihood for the underlying process

The log-likelihood for the underlying process measures a model's ability to describe the physical dynamics of the returns and realized measures (if applicable). We have outlined this log-likelihood for each model of interest:

- GARCH models

$$l_R = -T/2\log(2\pi) - 1/2\sum_t \log(h_t) - 1/2\sum_t (r_t - r - \lambda\sqrt{h_t} + 0.5h_t)^2/h_t$$

- Heston-Nandi GARCH models

$$l_R = -T/2\log(2\pi) - 1/2\sum_t \log(h_t) - 1/2\sum_t (r_t - r - (\lambda - 0.5)h_t)^2/h_t$$

- Realized GARCH model

$$l_R = -T/2\log(2\pi) - 1/2\sum_t \log(h_t) - 1/2\sum_t (r_t - r - (\lambda - 0.5)h_t)^2/h_t$$

$$l_{RV} = -T/2\log(2\pi\sigma^2) - 1/2\sum_t (\log x_t - \xi - \phi\log h_t - d_1 z_t - d_2(z_t^2 - 1))^2/\sigma^2$$

- GARV model

As the exact likelihood function for the volatility shock in the GARV model is difficult to obtain, the QMLE method is applied by assuming a bivariate normal distribution for (z_t, u_t) thus resulting in:

$$\mu_t = \begin{bmatrix} \mu_t^R \\ \mu_t^{RV} \end{bmatrix} = \begin{bmatrix} r + (\lambda - \frac{1}{2})\bar{h}_t \\ h_t^{RV} \end{bmatrix},$$
$$\Sigma_t = \begin{bmatrix} \bar{h}_t & -2\rho d_2\alpha_2\bar{h}_t \\ -2\rho d_2\alpha_2\bar{h}_t & 2\alpha_2^2(1 + 2d_2^2\bar{h}_t) \end{bmatrix}$$

Let $\mathbf{x}_t = (r_t, x_t)^T$:

$$\ell_{R,RV} = -T\log(2\pi) - \frac{1}{2}\sum_t \log(|\Sigma_t|) - \sum_t \frac{(\mathbf{x}_t - \mu_t)^T \Sigma_t^{-1}(\mathbf{x} - \mu_t)}{2}$$

- LHARG model

$$\ell_R = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_t \log(RV_t) - \frac{1}{2}\left\{\sum_t (r_t - r - (\lambda - \frac{1}{2})RV_t)^2/RV_t\right\}$$

[9]According to Vega's definition, this pricing error is an approximation of the error in the implied volatility..

$$\ell_{RV} = -\sum_t \left(\frac{RV_t}{\theta} + \Theta(\boldsymbol{RV_{t-1}}, \boldsymbol{L_{t-1}}) \right) + \sum_t \log \left(\sum_{k=0}^{\infty} \frac{RV_t^{\delta+k-1}}{\theta^{\delta+k}\Gamma(\delta+k)} \frac{\Theta(\boldsymbol{RV_{t-1}}, \boldsymbol{L_{t-1}})^k}{k!} \right)$$

In practice, we calculate the infinite sum up to 90 lags following the suggestion of Majewski, Bormetti, and Corsi (2015).

Log-likelihood for the pricing error

In this study, the option-pricing error of estimation is defined as the Vega-weighted pricing error, to mimic the difference in the implied volatility:

$$e_i = (O_i^{Mod} - O_i^{Mkt})/Vega_i$$

where O_i^{Mod} and O_i^{Mkt} are the model and market option price for the option i, respectively, assuming that the weighted pricing error follows the normal distribution of $N(0, \sigma_e^2)$ with the corresponding log-likelihood function:

$$l_o = -N/2 \log(2\pi\sigma_e^2) - 1/2 \sum_i e_i^2/\sigma_e^2$$

Since all 50 ETF options are European options, the implied volatility can be easily calculated by inverting the Black-Scholes formula with the corresponding option prices[10] In particular, O_i^{Mkt} is calculated using the mid-quote and O_i^{Mod} is calculated using the model price of the option i. The $Vega_i$ is calculated using the implied volatility derived from the O_i^{Mkt}.

Joint log-likelihood

The joint log-likelihood is constructed by adding the log-likelihood of the underlying and of the pricing errors together:

$$l = l_{R,RV} + l_o$$

where $l_{R,RV} = l_R + l_{RV}$ for the RGARCH and the LHARG models, while $l_{R,RV} = l_R$ for the models without realized measures.

V. Empirical results

The dataset

Our dataset contains data from 2015/02–2018/02; the following trimming process is applied:

(1) Options which do not satisfy the arbitrage restriction are dropped. As the option price is adjusted when the dividend is paid (i.e. dividend-protected), the SSE 50 ETF options can be treated as European options without dividends. The arbitrage restrictions are set as:

$$C(t) \geq \max(0, S(t) - Ke^{-rT}) \quad P(t) \geq \max(0, Ke^{-rT} - S(t))$$

(2) Options with zero trading volume are dropped.

(3) Options with maturities shorter than five or longer than 90 days are dropped. The resulting sub-sample represents up to 86% of the total trading volume.

(4) Options with low liquidity are dropped. This means that for every maturity on a given day, we drop those options with the trading volumes lower than the median volume of the group.

The resulting dataset contains 12,281 option prices. Table 2 provides an overview of the dataset with the number of prices and the average implied volatility. Panel A reports that, unlike the U.S. market, the Chinese market has a relatively balanced volatility smile rather than a volatility smirk. This balance indicates a relatively weaker leverage effect on the model. Panel B shows the volatility term structure which is downward sloping. In addition, most of the maturities are 60 days or less[11], indicating a lower demand for long-memory structures in the pricing model.

It is worth noting that the first year of trading experienced significant fluctuations and a particularly tight short-sell constraint due to the stock market bubble in mid-2015, the restarting of the IPOs in late 2015, and the 1 week of an implement of market circuit-breaker in the early 2016. Therefore,

[10]Although the implied volatility is unique given the option characteristics, it does not have a closed-form solution and it requires numerical method in order to retrieve it. In practice, we used the Dekker–Brent method (Press et al. 2002) to calculate implied volatility with the help of Matlab built-in function (blsimpv)..

[11]In our untrimmed dataset, options with maturities of less than 90 days account for 92% of the total trading volume.

Table 2. Option dataset summary.

	2015/02-2016/01	2016/02-2018/02	2015/02-2018/02
Total	3520	8761	12,281
	(0.366)	(0.162)	(0.221)
Panel A: Partitioned by moneyness			
S/K < 0.95	839	818	1657
	(0.405)	(0.203)	(0.305)
0.95 < S/K < 1.05	2041	7162	9203
	(0.343)	(0.151)	(0.194)
1.05 < S/K	640	781	1421
	(0.390)	(0.219)	(0.296)
Panel B: Partitioned by maturity			
DTM < 30	1672	3326	4998
	(0.374)	(0.159)	(0.231)
30 < DTM < 60	1507	3688	5195
	(0.360)	(0.164)	(0.221)
60 < DTM	341	1747	2088
	(0.357)	(0.164)	(0.196)
Panel C: Partitioned by VIX level			
VIX < 15		3620	3620
		(0.121)	(0.121)
15 < VIX < 30	503	4473	4976
	(0.259)	(0.173)	(0.182)
30 < VIX	3017	668	3685
	(0.384)	(0.308)	(0.370)

Note: The number of options in each category is provided. The average implied volatility is reported in parentheses.

we conduct our empirical investigation with both the full sample and the post-2015 sub-sample. We also use options from all trading days instead of only Wednesday due to the relatively limited strike prices available for the options.

Estimated parameters

Table 3 provides the parameter estimations for the different models using the full sample from 2015/02 to 2018/02. The first eight models are GARCH-type models and share the same notations, if applicable. The last one is the LHARG model and has a different set of parameters which we indicate accordingly.

Several commonly discussed features can be observed. First, all of the models have a highly persistent volatility process under both physical and risk-neutral measures. Second, most of the models have a positive and significant equity premium parameter (λ). The models with explicit volatility risk premium parameters (χ) indicate a higher risk-neutral volatility than their physical counterparts.

However, we also find an unconventionally positive leverage effect for most models (i.e. given the same magnitude of shocks, a positive shock induces higher volatility in the next period). This is likely to be explained by the market boom in early 2015 when the return and the volatility were highly positively correlated. If we estimate parameters without the data from 2015, only weak correlations are found in the GARCH models. Generally, the conventional leverage effect has been found to be weak in the Chinese market.

In-sample pricing performance

Table 4 provides the in-sample pricing performance across different models. The performance is evaluated by the mean-squared error of implied volatility (IVRMSE) defined as:

$$IVRMSE = \sqrt{\sum_{i=1}^{N} (IV_i^{Mkt} - IV_i^{Mod})^2 / N} \times 100$$

where IV_i^{Mod} and IV_i^{Mkt} are the implied volatilities calculated from the model price and the market price, respectively. Unlike the option price-based RMSE, the IVRMSE is a standardized pricing error; this means that it avoids the great weight assigned to high-price options. Table 4 provides a summary of the pricing performance with decomposed details. Bold numbers indicate the minimum IVRMSE for each row.

For the overall performance, the total IVRMSE shows that models with realized measures generally perform better than those without realized measures. The GARV model delivers the best fit (with a 21% IVRMSE reduction compared to the HNG),

Table 3. Full sample parameter estimation: 2015/02-2018/02.

	GARCH	GJR	NG	EG	HNG	HNGvd	GARV	RGARCH		LHARG
λ	−0.0353	0.2723	0.2057	0.0849	1.0950	1.1356	20.4439	0.0574	λ	6.6784
	(0.0361)	(0.0519)	(0.0299)	(0.0005)	(0.4612)	(0.8032)	(1.7763)	(0.0762)		(3.5198)
β	0.9080	0.9011	0.9161	0.9874	0.9867	0.9867	0.9818	0.9954		
	(0.0027)	(0.0105)	(0.0114)	(0.0001)	(0.0010)	(0.0020)	(0.0011)	(0.0010)		
τ_1	0.0781	0.1299	0.0702	0.0354	7.91E-06	6.95E-06	2.07E-06	−0.0220	θ	8.20E-05
	(0.0027)	(0.0137)	(0.0113)	(0.0012)	(2.51E-07)	(5.12E-07)	(1.93E-07)	(0.0045)		(8.51E-06)
τ_2		−0.0708	−0.3372	0.1754	−23.8340	−25.4704	−61.4457	0.0460	δ	1.4055
		(0.0064)	(0.0442)	(0.0063)	(3.9761)	(4.1017)	(2.6017)	(0.0092)		(0.0879)
γ							0.3767	0.1085	$\theta\beta_d$	0.4928
							(0.0234)	(0.0217)		(0.0547)
κ							0.2105		$\theta\beta_w$	0.2648
							(0.0244)			(0.0302)
ξ								2.2893	$\theta\beta_m$	0.0719
								(0.2441)		(0.0068)
ϕ							2.89E-06	1.3024	$\theta\beta_q$	0.0494
							(2.70E-06)	(0.0291)		(0.0042)
d_1							8.14E-06	0.0032	$\theta\beta_y$	0.0580
							(2.22E-06)	(0.0092)		(0.0031)
d_2							371.1036	0.1924	α	2.35E-05
							(50.7164)	(0.0512)		(4.59E-06)
ρ/σ							0.0451	0.5817	γ	23.4299
							(0.0449)	(0.0483)		(7.3757)
χ						1.0670	9.1939	0.0379	χ	356.6782
						(0.0537)	(1.8250)	(0.1472)		(559.7493)
$\log(\tilde{h})$	−8.5487	−7.4071	−7.7362	−7.7456	−7.5059	−7.5695	−7.8923	−8.2587	$\log(\tilde{h})$	−8.5758
	(0.0216)	(0.1134)	(0.0758)	(0.0210)	(0.0593)	(0.0639)	(0.0839)	(0.5922)		(0.5952)
π^P	0.9861	0.9956	0.9943	0.9874	0.9912	0.9912	0.9835	0.9954	π^P	0.9369
π^Q	0.9862	0.9870	0.9875	0.9874	0.9908	0.9908	0.9826	0.9954	π^Q	0.9999
ℓ	18,800.2	19,034.1	18,820.5	19,830.6	18,655.6	18,656.4	26,618.3	19,887.4	ℓ	269,104

Note: Here we report $\theta\beta_i$ and $\theta\alpha_i$ instead of β_i and α_i for the LHARG model to make it easier to compare different models. The robust standard errors are reported in parentheses. The variables π^P and π^Q are persistence parameters under physical and risk-neutral measures, respectively. The variable ℓ is the log-likelihood value. The row "ρ/σ" reports parameter ρ for the GARV model and σ for the RGARCH model.

Table 4. Full sample pricing performance: 2015/02-2018/02.

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	12.2482	6.3602	6.2441	6.3489	5.8701	6.5301	6.5306	5.6821	**5.1600**	5.5718
Panel A: Partitioned by time period										
2015/02-2016/01	18.4495	10.3053	9.9942	10.2898	9.3670	10.9770	10.9758	9.2509	**8.3374**	8.7830
2016/02-2018/02	8.5751	3.7338	3.7949	3.7197	3.6002	3.3862	3.3896	3.2815	**3.0657**	3.5338
Panel B: Evaluation by moneyness										
S/K < 0.95	16.6198	8.1000	7.7831	8.0653	7.3916	9.4044	9.4021	7.4116	**6.7632**	7.0091
0.95 < S/K < 1.05	10.9419	5.7206	5.6223	5.6690	5.3168	5.7390	5.7392	5.1939	**4.7354**	5.0488
1.05 < S/K	14.1084	7.9536	7.9874	8.2021	7.2333	7.4112	7.4166	6.3990	**5.6849**	6.8420
Panel C: Evaluation by maturity										
DTM < 30	13.5382	6.8011	6.6268	6.7727	6.4178	7.4022	7.4017	6.0932	**5.5587**	5.9368
30 < DTM < 60	11.8430	6.3989	6.2846	6.3709	5.7881	6.2062	6.2082	5.7734	**5.2115**	5.6348
60 < DTM	9.7303	5.0033	5.0684	5.0951	4.5440	4.8767	4.8759	4.2307	**3.8886**	4.3753
Panel D: Evaluation by VIX level										
VIX < 15	10.3012	2.8344	2.7746	2.7710	2.8078	2.8551	2.8558	2.4064	**2.2971**	2.7943
15 < VIX < 30	6.3190	3.7128	3.7503	3.6475	3.5808	3.6011	3.6087	3.2686	**3.1925**	3.4713
30 < VIX	18.4879	10.3817	10.1403	10.4008	9.4530	10.8107	10.8084	9.3333	**8.3533**	8.9051

Note: The bold numbers indicate the minimum IVRMSE values in each row.

followed by the LHARG and RGARCH models. The non-affine GARCH models (GARCH/GJR/NG/EG) are generally better than the affine GARCH models (HNG/HNGvd). A similar pattern is documented by Christoffersen, Jacobs, and Ornthanalai (2013b). As the leverage effects are weak, there is no significant performance difference between symmetric (GARCH) and asymmetric models (GJR/NG).

For the decomposed performance, we isolate the 2015 sub-sample from the dataset and price it using the parameters estimated from the full sample. The results clearly demonstrate that, in the first year of the SSE 50ETF options, the behaviour was significantly

Table 5. Pricing performance using different realized measures.

	LHARG	GARV	RGARCH
RV1min	5.8020	**5.0863**	5.6310
RV5min	**5.7282**	5.1600	5.5718
RV10min	5.9253	5.2245	**5.5281**
RV30min	5.9372	5.3567	5.5434
RK	5.9089	5.2841	5.6233
BV	5.8281	5.1562	5.5694
TSRV	5.9553	5.2847	5.6172

Note: This table reports the full sample pricing performances (IVRMSE) of three high-frequency data-based option pricing models (GARV/LHARG/RGARCH) using different realized measures. We consider a variety of classes of estimators for the asset price volatility, including the realized variance (RV1min, RV5min, RV10min and RV30min) from AB1998, the two-scale realized variance (TSRV) from zhang2005tale, the realized kernel (RK) from BHLS2008, and the bi-power variation (BPV) from BN2004. The bold numbers indicate the minimum IVRMSE values in each column.

different than in the following years; the full sample parameters show extremely large IVRMSEs for the first year, which then drop sharply when the data for first-year are eliminated. The GARV model remains the best model for all of the sub-cases. Moreover, the log-linear models perform better when the volatility level is high and the option is deep out-of-the-money.

As the last three models rely on realized measures, we also provide performance comparisons for a range of realized measures. Table 5 reports the IVRMSE for the three models using different realized measures as well as the realized variance for different sampling frequencies[12] of Andersen and Bollerslev (1998), the two-scale realized variance (TSRV) of Zhang, Mykland, and At-Sahalia (2005), the realized kernel (RK) of Barndorff-Nielsen et al. (2008), and the bi-power variation (BPV) of Barndorff-Nielsen (2004). The bold numbers indicate the minimum IVRMSE for each column. Interestingly, unlike the results based on volatility forecasting, the complex realized measures do not generally provide better option prices. Traditional-realized variance with sampling intervals as short as 10 min leads to a reasonably good performance.

We also provide Black-Scholes (BS) results with the volatility being calibrated from the full sample. Given the dramatic changes in volatility levels between 2015 and 2018, it is not surprising that the BS model delivers the worst pricing performance.

Out-of-sample pricing performance

To incorporate the realized measures, the number of parameters is significantly increased for the LHARG, GARV and RGARCH models. It is important to check that the superior performance of these models is not merely due to in-sample overfitting. In the literature, three major out-of-sample evaluation procedures have been proposed. The first one estimates the parameters using data from the first several years and then use them to value option prices for the following years (e.g. Christoffersen and Jacobs 2004). The second one uses a rolling-window framework in which the parameters are updated once in each time period (e.g. Christoffersen and Diebold 2006). The third one splits the sample into Wednesday (for parameter estimation) and Thursday (for pricing evaluation) sub-samples within the same time period (e.g. Christoffersen, Jacobs, and Minouni 2010). As the Chinese data cover a much shorter period and are more volatile than the U.S. data, we use the rolling-window framework as our primary method and the split sample method as a robustness check[13]

The evaluation of the out-of-sample pricing performance is based on a rolling window of 252 trading days, with the parameters updated on a monthly basis. We evaluate the out-of-sample pricing errors from 2016/02 to 2018/02; the observations from 2015/02 to 2016/01 are used as a pre-sample to determine the first parameter for the out-of-sample analysis. The results are presented in Table 6 and include the decomposed results related to different moneyness, maturity and VIX level.

Similar to the in-sample results, the models with realized measures have better out-of-sample pricing performance. The GARV model still generates the smallest total pricing error, but the decomposed results are mixed for the three models. The performance gain of the leverage GARCH models over the standard GARCH model is not significantly larger. The HNGvd model delivers results similar to the standard HNG model. In short, the results of the rolling-window method for examining the out-of-sample data suggest that the performance gain of

[12]RV1min, RV5min, RV10min and RV30min are considered.

[13]Given the much smaller sample we have, the splitting method in this study estimates parameters using Monday/Wednesday/Friday data and evaluates option prices using Tuesday/Thursday data. We use MWF/TTh to denote this method..

Table 6. Out-of-sample (rolling-window) pricing performance: 2016/02–2018/02.

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	9.0341	4.1551	3.9833	3.9705	3.7481	3.9247	3.8531	3.3378	**3.2324**	3.3341
Panel A: Evaluation by moneyness										
S/K < 0.95	9.6641	4.1501	3.9875	3.7276	4.1867	4.2714	3.9163	**3.1152**	3.2558	3.1925
0.95 < S/K < 1.05	9.0475	4.0609	3.8939	3.8966	3.6008	3.8526	3.8267	3.2987	**3.2180**	3.3191
1.05 < S/K	8.1892	5.0109	4.7927	4.8772	4.6104	4.2265	4.0376	3.9610	**3.3449**	3.6215
Panel B: Evaluation by maturity										
DTM < 30	9.3175	3.9309	3.8122	3.8930	3.6241	3.9890	3.9886	**3.1089**	3.2063	3.4035
30 < DTM < 60	9.3756	4.4317	4.2598	4.2098	3.9133	4.1514	4.0625	3.4520	**3.3850**	3.4985
60 < DTM	7.6447	3.9566	3.6792	3.5689	3.6183	3.2519	3.0531	3.5131	2.9351	**2.8008**
Panel C: Evaluation by VIX level										
VIX < 15	5.8761	2.4747	2.4760	2.5273	2.5463	2.4859	2.4851	2.2456	**2.2443**	2.3770
15 < VIX < 30	7.4012	4.7196	4.4049	4.3748	3.7650	4.2332	4.0983	3.4616	3.3632	**3.2405**
30 < VIX	12.8435	6.6072	6.6846	6.6018	7.3254	6.9726	6.9760	6.1652	**5.8145**	6.6765

Note: The out-of-sample pricing performance evaluation is based on a rolling window of 252 trading days, with the parameters updated on a monthly basis. We evaluate the out-of-sample pricing errors using observations from the 2016/02 to 2018/02 period and use observations from the2015/02 to 2016/01 period as a pre-sample to obtain the first parameter for the out-of-sample analysis. The bold numbers represent the minimum IVRMSE value in each row. BS stands for the classic Black-Scholes model where volatility is estimated by minimizing the mean squared Vega for the weighted pricing error.

Table 7. Out-of-sample pricing performance (MWF/TTh): 2016/02-2018/02.

	BS	GARCH	GJR	NG	EG	HNG	HNGvd	LHARG	GARV	RGARCH
Total IVRMSE	6.1244	3.0666	3.0691	3.0747	3.0794	3.1600	3.1526	**2.7796**	2.7805	2.8846
Panel A: Partitioned by moneyness										
S/K < 0.95	7.3434	2.7994	3.3350	2.7942	3.0241	3.9588	3.9483	**2.5132**	3.0451	3.0243
0.95 < S/K < 1.05	5.4585	2.8200	2.8669	2.8280	2.8371	2.9539	2.9489	**2.6092**	2.6207	2.7582
1.05 < S/K	9.5211	5.2529	4.4278	5.3003	5.0030	4.0116	3.9996	4.3444	**3.8257**	3.8380
Panel B: Partitioned by maturity										
DTM < 30	5.9490	2.8301	2.8968	2.8324	2.9331	3.1090	3.0971	**2.7019**	2.8732	2.8812
30 < DTM < 60	6.5814	3.2018	3.1411	3.2015	3.1669	3.3095	3.2842	2.8546	**2.8085**	2.9243
60 < DTM	5.4501	3.2461	3.2564	3.2838	3.1876	2.9360	2.9801	2.7777	**2.5149**	2.8063
Panel C: Partitioned by VIX level										
VIX < 15	4.9164	2.2558	2.2723	2.2553	2.3446	2.4316	2.4243	**1.9796**	2.0578	2.2127
15 < VIX < 30	4.3317	3.0044	3.0451	3.0062	2.9891	3.0626	3.0532	2.6264	**2.5910**	2.7669
30 < VIX	15.2485	5.8826	5.7432	5.9286	5.8402	5.9633	5.9568	5.7518	5.7211	**5.5395**

Note: This table reports the out-of-sample pricing performance (IVRMSE) for the 2016/02 to 2018/02 period. We estimate the parameters usingMonday/Wednesday/Friday data from the 2016/02 to 2018/02 period. Keeping the parameters fixed, we value the Tuesday/Thursday options within the same time period. The bold numbers indicate the minimum IVRMSE value in each row.

option-pricing models that are based on realized measures is not due to in-sample overfitting.

For the split sample method, we estimate the parameters using Monday/Wednesday/Friday data from 2016/02 to 2018/02 period and value the Tuesday/Thursday options within the same time span[14] The results are presented in Table 7. Most of the results are the same as when the rolling-window method is used.

VI. Conclusion

The existing literature on volatility models for the Chinese stock market typically focuses on the fit and forecast of the volatility series itself. This study, on the other hand, provides a comparison of discrete-time volatility models through the pricing of SSE 50 ETF options. The models discussed in this article including both affine and non-affine models, as well as state-of-the-art models with newly developed realized measures. Since volatility takes a central position in the pricing of options, our results contribute both to the literature on volatility comparisons and that on option pricing. That may therefore be relevant to both academics and practitioners.

In line with the current literature, we found that the non-affine structure is superior to affine models when it comes to modelling the volatility surface. We also found that models based on realized measures have a significantly greater pricing-performance gain when compared to models based on daily returns across measures such as the volatility surface and across different volatility regimes. Out-of-sample

[14]The sample is trimmed to be comparable to the rolling-window results.

comparisons confirmed that such gains are not simply due to increasing parameters.

Most of the literature on pricing options with high-frequency data has only focused on single-realized measures; we add to this literature by checking the pricing difference using a list of commonly-utilized realized measures. Surprisingly, complicated-realized measures do not provide better pricing results when compared to simple-realized measures, such as the conventional-realized variance. This is, however, not the case for the forecast of volatility series itself, where complicated measures such as realized kernel (Barndorff-Nielsen et al. 2008), two-scale realized variance (Zhang, Mykland, and At-Sahalia 2005) appear to be empirically appealing.

Another interesting result is that the well-documented leverage effect embedded in the option price is weak for the Chinese option market, which contradicts its previously documented effect for the U.S. market. Our analysis also indicated significant differences between the first trading year (i.e. 2015) and subsequent trading years. These differences are possibly due to the extreme market volatility and severe limits imposed on arbitrage. This seems to have been overlooked, as most existing research on the Chinese option market is either based on 2015 only (e.g. Li et al. 2018) or is based on a mixed dataset (e.g. Yang 2018). It is therefore necessary to analyse the data from this year separately when investigating the Chinese option market.

Future studies should address several issues. The first is the modelling of jumps in the underlying process. Christoffersen, Jacobs, and Ornthanalai (2012) highlighted the importance of dynamic jump intensities in option pricing. It is natural to expect that such a feature might also be important in the Chinese market, especially when realized jumps are involved. The second issue is the need for comparisons between discrete-time models and continuous-time models, especially with regard to continuous-time models with realized measures.

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References

Andersen, T. G., and T. Bollerslev. 1998. "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts." *International Economic Review* 39: 885–905.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys. 2003. "Modeling and Forecasting Realized Volatility." *Econometrica* 71: 579–625.

Bakshi, G., C. Cao, and Z. Chen. 1997. "Empirical Performance of Alternative Option Pricing Models." *The Journal of Finance* 52: 2003–2049.

Barndorff-Nielsen, O. E. 2004. "Power and Bipower Variation with Stochastic Volatility and Jumps." *Journal of Financial Econometrics* 2: 1–37.

Barndorff-Nielsen, O. E., P. R. Hansen, A. Lunde, and N. Shephard. 2008. "Designing Realized Kernels to Measure the Ex Post Variation of Equity Prices in the Presence of Noise." *Econometrica* 76: 1481–1536.

Bilson, J. F., S. B. Kang, and H. Luo. 2015. "The Term Structure of Implied Dividend Yields and Expected Returns." *Economics Letters* 128: 9–13.

Black, F., and M. S. Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81: 637–654.

Chernov, M., and E. Ghysels. 2000. "A Study Towards A Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Options Valuation." *Journal of Financial Economics* 56: 407–458.

Christoffersen, P., and F. Diebold. 2006. "Financial Asset Returns, Direction-of-Change Forecasting, and Volatility Dynamics." *Management Science* 52 (8): 1273–1287.

Christoffersen, P., B. Feunou, K. Jacobs, and N. Meddahi. 2014. "The Economic Value of Realized Volatility: Using High-Frequency Returns for Option Valuation." *Journal of Financial and Quantitative Analysis* 49 (3): 663–697.

Christoffersen, P., S. L. Heston, and K. Jacobs. 2013a. "Capturing Option Anomalies with a Variance-Dependent Pricing Kernel." *Review of Financial Studies* 26 (8): 1963–2006.

Christoffersen, P., and K. Jacobs. 2004. "Which GARCH Model for Option Valuation?" *Management Science* 50 (9): 1204–1221.

Christoffersen, P., K. Jacobs, and K. Minouni. 2010. "Volatility Dynamics for the S&P500: Evidence from Realized Volatility, Daily Returns, and Option Prices." *Review of Financial Studies* 23 (8): 3141–3189.

Christoffersen, P., K. Jacobs, and C. Ornthanalai. 2012. "Dynamic Jump Intensities and Risk Premiums: Evidence from S&P500 Returns and Options." *Journal of Financial Economics* 106: 447–472.

Christoffersen, P., K. Jacobs, and C. Ornthanalai. 2013b. "GARCH Option Valuation: Theory and Evidence." *Journal of Derivatives* 21: 8–41.

Corsi, F. 2009. "A Simple Approximate Long-Memory Model of Realized Volatility." *Journal of Financial Econometrics* 7: 174–196.

Duan, J.-C. 1995. "The GARCH Option Pricing Model." *Mathematical Finance* 5 (1): 13–32.

Duan, J., G. Gauthier, and J. Simonato. 1999. "An Analytical Approximation for the GARCH Option Pricing Model." *Journal of Computational Finance* 2: 75–116.

Engle, R. F., and T. Bollerslev. 1986. "Modelling the Persistence in Conditional Variances." *Econometric Reviews* 5: 1–50.

Engle, R. F., and G. M. Gallo. 2006. "A Multiple Indicators Model for Volatility Using Intra-daily Data." *Journal of Econometrics* 131: 3–27.

Engle, R. F., and V. K. Ng. 1993. "Measuring and Testing the Impact of News on Volatility." *The Journal of Finance* 48: 1749–1778.

Eraker, B. 2004. "Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices." *The Journal of Finance* 59: 1367–1403.

Glosten, L. R., R. Jagannathan, and D. E. Runkle. 1993. "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance* 48: 1779–1801.

Hansen, P. R., and Z. Huang. 2016. "Exponential GARCH Modeling with Realized Measures of Volatility." *Journal of Business & Economic Statistics* 34 (2): 269–287.

Hansen, P. R., Z. Huang, and H. H. Shek. 2012. "Realized GARCH: A Joint Model of Returns and Realized Measures of Volatility." *Journal of Applied Econometrics* 27 (6): 877–906.

Heston, S. L. 1993. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *Review of Financial Studies* 6: 327–343.

Heston, S. L., and S. Nandi. 2000. "A Closed-Form GARCH Option Valuation Model." *Review of Financial Studies* 13: 585–625.

Hsieh, K. C., and P. Ritchken. 2005. "An Empirical Comparison of GARCH Option Pricing Models." *Review of Derivatives Research* 8: 129–150.

Huang, X. ,., J. Liu,, X. Zhang,, and Y. Zhu. 2018. *Volatility Premium and Term Structure of China Blue-Chip Index Options*. Emerging Markets Finance and Trade. 1–16.

Huang, Z., C. Tong, and T. Wang. 2019. "VIX Term Structure and VIX Futures Pricing with Realized Volatility." *Journal of Futures Markets* 39: 72–93.

Huang, Z., T. Wang, and P. Hansen. 2017. "Option Pricing with the Realized GARCH Model: An Analytical Approximation Approach." *Journal of Futures Markets* 37: 3.

Kou, S. G., and H. Wang. 2004. "Option Pricing under a Double Exponential Jump Diffusion Model." *Management Science* 50: 1178–1192.

Li, J., Y. Yao, Y. Chen, and C.-F. Lee. 2018. "Option Prices and Stock Market Momentum: Evidence from China." *Quantitative Finance* 18: 1517–1529.

Majewski, A., G. Bormetti, and F. Corsi. 2015. "Smile from the Past: A General Option Pricing Framework with Multiple Volatility and Leverage Components." *Journal of Econometrics* 187 (2): 521–531.

Nelson, D. B. 1991. "Conditional Heteroskedasticity in Asset Returns: A New Approach." *Econometrica* 59: 347–370.

Ofek, E., M. Richardson, and R. F. Whitelaw. 2004. "Limited Arbitrage and Short Sales Restrictions: Evidence from the Options Markets." *Journal of Financial Economics* 74: 305–342.

Press,, W. H. ,., S. A. Teukolsky,, W. T. Vetterling,, and B. P. Flannery. 2002. "Root Finding and Nonlinear Sets of Equations." In *Numerical Recipes in C++: The Art of Scientific Computing*, 363–365. *2nd* ed.

Shephard, N., and K. Sheppard. 2010. "Realising the Future: Forecasting with High Frequency Based Volatility (HEAVY) Models." *Journal of Applied Econometrics* 25 (2): 197–231.

Stentoft, L. 2011. "American Option Pricing with Discrete and Continuous Time Models: An Empirical Comparison." *Journal of Empirical Finance* 18: 880–902.

Wang, J., S. Chen., Q. Tao., and T. Zhang. 2017a. "Modelling the Implied Volatility Surface Based on Shanghai 50ETF Options." *Economic Modelling* 64: 295–301.

Wang, T., Y. Shen, Y. Jiang, and Z. Huang. 2017b. "Pricing the CBOE VIX Futures with the Heston–Nandi GARCH Model." *Journal of Futures Markets* 37: 641–659.

Xiong, W., and J. Yu. 2011. "The Chinese Warrants Bubble." *American Economic Review* 101: 2723–2753.

Yang, X. 2018. "Good Jump, Bad Jump, and Option Valuation." *Journal of Futures Markets* 38: 1097–1125.

Yue, T., S. Gehricke, J. E. Zhang, and Z. Pan. 2019. "How Do Chinese Option-Traders "smirk" on China: Evidence from SSE 50 ETF Options." *Working Paper*.

Zhang, L., P. A. Mykland, and Y. At-Sahalia. 2005. "A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-frequency Data." *Journal of the American Statistical Association* 100: 1394–1411.