

Lake Victoria Fish Stocks and the Effects of Water Hyacinths on the Catchability of Fish

Eseza Kateregga and Thomas Sterner

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Abstract

This paper analyses the effects of the invasion of water hyacinths on fishing in Lake Victoria. We built two fairly standard Schaefer-type models that have one innovation: they allow the water hyacinth abundance to affect catchability. We estimated static and dynamic CPUE (catch per unit of effort) functions for Lake Victoria fisheries. We investigated the trend in the lake's stocks during the period 1983–2000 and focused particularly on the effect of the water hyacinth on fish stocks and on catchability coefficients. The results shows that while fish stocks have fallen since 1990, this decline appears to have been at least temporarily halted by the declining catchability of fish due to the growing abundance of water hyacinths. The impact of the hyacinth on the catchability of fish was greatest in the Kenyan section of Lake Victoria. Although the hyacinths had many negative effects, one important effect has been to effectively hinder fishing and, thereby—paradoxically—stop or at least postpone serious overfishing.

Key Words: Water hyacinth, catchability coefficients, Lake Victoria fish stocks

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Introduction

The presence of the water hyacinth in Lake Victoria has been cited as far back as 1989 (Moorhouse et al. 2001). By early 1990s, the adverse effects arising from water hyacinth mats were alarming. (See Twongo and Balirwa 1995; or LVEMP 1997.) Water hyacinth mats invaded fishing grounds and blocked waterways. For the individual fisherman, the hyacinth mats reduced their catch by covering fishing grounds, delaying access to markets due to loss of output, increasing fishing costs due to the time and effort spent clearing waterways, forcing translocation, and causing loss of nets. Mailu's (2001) report cited declines of 14 percent, 37 percent, and 59 percent in the catches of *Oreochromis* (a large genus of tilapia), *Clarias* (a genus of catfish), and *Mormyrus* (a genus of bottom-feeding breams), respectively, in the Kenyan section of the lake. Twongo (1998) noted that the weed mats sealed off breeding, nursery, feeding, and fishing grounds for various inshore fish species, like tilapia and young Nile perch. The mats also had detrimental effects by blocking light, severely reducing oxygen levels, and allowing poisonous gases, such as ammonia and hydrogen sulfide, to accumulate.

At the same time, the water hyacinth is believed to have promoted fish diversity, particularly smaller species and the young. Mechanisms for this include providing shelter from predators as well as reducing fishing pressure. It enhanced the abundance of lungfish and *Haplochromines* (riverine "haps") and depressed the number of tilapias and *Synodontis*, a member of the catfish genus (Twongo 1998). Thus, structural changes in the species composition of Lake Victoria's fish stocks may have been induced by the water hyacinth infestation of the lake. Evidence of the reappearance of certain species that had been declared extinct in the last 15

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years has been acknowledged in the three sections of the lake. However, the overall biological effect of the weed on aggregate fish stocks and catches is not well known.

Data deficiencies make analysis of the weed effect on fish stock composition dynamics rather difficult. Efforts to clear the invasive weed and to understand its effects have been hampered in the past by a lack of knowledge concerning the extent of the infestation. In an earlier work (2007), we put considerable effort into developing an index of water hyacinth abundancy. The purpose of this study, however, was to explore the effects that water hyacinths have on fishing. There has been little research on this topic to date. However, a report on the social, economic, and environmental impacts of the water hyacinth in the Lake Victoria basin (Mailu 2001) noted that fish catch increased during the time that the Kenya section of the lake was heavily infested by the weed. (We are not aware of any mention of such effects for the Tanzania and Uganda sections.)

Availability of data was severely limited, and we had to look at time-series data on aggregate catches, the number of vessels engaged in fishing on the lake, and the abundance of the water hyacinth over the period 1968–2000 in order to study the impact of the water hyacinth on the fish catchability coefficients. The methodological innovation proposed here is to make the catchability coefficient a function of the water hyacinth level. This enabled us to estimate static catch-effort functions. The same approach was then introduced in a model where the fish stock growth function evolved as a first order difference equation. This enabled us to estimate dynamic CPUE functions. The parameters for the sectional fisheries were derived from the results estimated in both the static and dynamic functions. We used the results and the water hyacinth abundance to calculate the water hyacinth-induced catchability coefficients.

The paper is organized into seven additional sections. Section 2 reviews fisheries modeling. The theoretical framework behind the analyses is presented in section 3. Estimation issues are discussed in section 4, and the data provided in section 5. The estimation procedure is discussed in section 6, and the results from the estimated functions are presented in section 7. The conclusions from the analyses are presented in section 8.

2. Fisheries Modeling: A Review

The analysis of fish population dynamics attempts to make predictions about the birth, death, growth, and movement processes of fish (Hilborn and Walters 1992). Quantitative models of fish population include simple models that consider only the biomass of the population (surplus production models) and the explicit age-structured models with more detailed

population processes, such as growth and recruitment (Clark 1990; Hilborn and Walters 1992; Hannesson 1993). These are often adjusted to incorporate the impact of external effects, such as predation, competition for food supply, and other environmental factors (Flaaten 1991).

The first class of models are non-age-structured models. They require little data and are based on the following assumptions: (1) fish population dynamics are regulated by density-dependent growth; (2) the effects of the age structure are negligible; (3) time delays in production processes are not considered; (4) biomass is homogeneously distributed in the area; (5) there is a balance between immigration and emigration rates; and (6) catchability rates and fishing patterns are constant (Arnason et al. 1995).¹ The second class of models is based on natural equations for an age-structured population and, unlike the previous class, it can make use of auxiliary information from biological studies. The incorporation of the behavior of fishermen and fleet dynamics with the fish population dynamics enables the researcher to capture the yield-effort relationship from the observed data.

The models used in our analysis subscribed to the first group of the categories above. The Schaefer (1957) model and its variants have been widely applied in fisheries analyses. (See, for example, Bell 1972; Smith 1977; Hannesson 1983.) Despite frequent application, however, the model has been criticized on several grounds.

To fisheries biologists, lumping together a number of species of different sizes and age-structures into a uniform stock is implausible. Further, the logistic growth curve has been found inappropriate for many fish species. More specifically, it has been observed that at high stock levels, fish growth may exceed what is implied by the points on a logistic curve (Clark 1990). Economists have criticized the model for its inadequacy in incorporating fishing costs. Also, the omission of the effect of the discount rate on the fishermen's inter-temporal decision making implies that the dynamics are not well analyzed. Other issues of concern include the assumption of constant growth parameters, the inability to explore the interaction between growth and catch, the tendency to ignore differences in growth and catch rates due to variations in the size distribution of fish, and the assumption of a steady state in the fishery.

In a bid to improve the models, several modifications have been pursued, including these variants of the Schaefer model: the Fox model which employs a Gompertz growth function;

¹ Density-dependent growth means that the future growth rate of fish stock depends on the size of the current stock.

Hall's (1977) introduction of an environmental factor (temperature) in the growth function in order to improve on Bell's (1972) estimates of the U.S. lobster fishery; and Smith's (1980) replacement of the steady-state assumption with a dynamic stochastic model. This is a formulation in which the fish-stock growth function evolves as a first order difference equation with a stochastic error; Townsend (1986) replaced the linear catch-rate equation with a quadratic yield effort function.

The use of a surplus production model in which fish stock growth is specified using finite difference equations to assess the stock in Lake Victoria is not novel. Pitcher and Bundy (1995) use the yield-per-recruit model and the modified Schaefer model in which the growth function takes a finite difference equation form. They assessed the Nile perch fish stocks in Lake Victoria for the period 1979–90.

In our study, data availability forced us to adopt fairly simple Schaeffer-type models and we did model aggregate stocks. Consequently, the estimated parameters may at best be indicators of the average values for the various species, and results must be interpreted with caution. The main novelty in this paper derives from the incorporation of the water hyacinth variable in the fish harvest function. We relaxed the assumption of a constant catchability coefficient and let it vary with the level of water hyacinth abundance. We also did not assume equilibrium in one of our models.

3. Conceptual Framework

The conceptual framework of the fisheries models that are estimated is presented in this section. We explored the effect of the water hyacinth on the catchability coefficient function by using the Schaefer fisheries model and a modified variant of the Schaefer model in which the stock growth function is a first order difference equation, and where the equilibrium assumptions postulated in the static approach are relaxed.

3.1 *A Static Schaefer Model with Weed Density Dependent Catchability*

This model is built on three premises: (1) that catch per unit of effort is directly proportional to the density of fish; (2) that the density of fish is directly proportional to the abundance of fish at time t ; and (3) that the catchability coefficient depends on the abundance of water hyacinth in the lake in period t . The third assumption deviates from the original version of the Schaefer model in which the catchability coefficient is assumed to be constant. The fishery dynamics are presented by the fishery production function.

$$\frac{dB}{dt} = \dot{B} = rB_t \left(1 - \frac{B_t}{K}\right) - h_t \quad (1)$$

Fish growth takes a logistic function form. Water hyacinth mats were observed to have both positive and deleterious effects on fish growth, although the net effects are still unknown. For this reason, we omitted the hyacinth effect in the fish growth function.

In the equation below, h_t is the fish harvest function for the period t . The harvest of fish is a function of effort water hyacinth abundance and the fish stocks. The fisheries production function is assumed to be linear in effort and fish stocks, and is presented as:

$$h_t = q_t(W_t)E_tB_t, \quad (2)$$

$$\text{where we assume } \frac{\partial h}{\partial W} = \frac{\partial h}{\partial q} \cdot \frac{\partial q}{\partial W} \leq 0, \quad \frac{\partial h}{\partial E} > 0, \quad \frac{\partial h}{\partial B} > 0, \quad (3)$$

and where W_t is the water hyacinth abundance in period t and measured in hectares of hyacinth mats, E_t is the level of effort in period t , and B_t is the fish stock in period t . $q(W)$ is the water hyacinth-induced catchability coefficient, which is assumed to be a function of water hyacinth abundance and other factors. The water hyacinth coverage of the fishing grounds reduces the ease with which fish is harvested. Therefore, its effect on the coefficient and on harvests is expected to be negative. We defined the relation between q and W as (4):

$$q_t(W_t) = \alpha \left(1 - \frac{W_t}{Z}\right), \quad (4)$$

where Z is defined as the area of the lake that is relevant for fishing operations and is measured in hectares. $\left(1 - \frac{W_t}{Z}\right)$ is the proportion of the fishing area which is free of the water hyacinth mats. (See, further, Kateregga and Sterner 2007.) The effect of the water hyacinth on the catchability of fish is then given by:

$$\frac{dq}{dW} \begin{cases} = -\frac{\alpha}{Z} & \text{if } \underline{W} \leq W_t \leq \bar{W} \\ = 0 & \text{if } 0 \leq W_t < \underline{W} \end{cases}, \quad (5)$$

where $W \in (0, \bar{W})$ and $\alpha \in (0, 1)$. $\underline{W}$ is the minimum amount of hyacinth coverage necessary for inducing inconveniences in the fishable zone Z . $\bar{W}$ is the maximum coverage by the water hyacinth mats that may be realized without weed control interventions in zone Z . α corresponds to the exogenous component of the catchability coefficient.

Ricker (1975) defined the fish catchability coefficient as the fraction of a fish stock that is caught by a defined unit of fishing effort. This implies that $0 \leq q \leq 1$. Periods characterized by the absence of water hyacinth-related interruptions in fishing are assumed to be those with either very low levels of hyacinth matting or those periods before the lake was infested with the weed. In these instances, the value of W_t is less than $\underline{W}$. The water hyacinth-induced catchability coefficient then reduces to the constant in the Schaefer model, that is, $q = \alpha$. The net effect of hyacinths on stocks, as we discussed above, is not yet known. Using (4) to substitute for q in (2), the harvest function can be rewritten as:

$$h_t = \alpha \left(1 - \frac{W_t}{Z}\right) E_t B_t, \quad (6)$$

and:

$$\frac{dh}{dW} \begin{cases} = -\frac{\alpha}{Z} E_t B_t & \text{if } \underline{W} \leq W_t \leq \bar{W} \\ = 0 & \text{if } 0 \leq W_t < \underline{W} \end{cases}. \quad (7)$$

When the fishery is in equilibrium, fish harvests are always equal to the sustainable yield for some defined level of stocks (Hannesson 1993). Making use of this condition, (1) and (6) imply that the catch-effort (or the sustainable yield) function is given by:²

$$h_t = A \left(1 - \frac{W_t}{Z}\right) E_t - D \left(1 - \frac{W_t}{Z}\right)^2 E_t^2, \quad (8)$$

where $A = \alpha K$, and $D = \frac{\alpha^2 K}{r}$. $\left(1 - \frac{W_t}{Z}\right) E_t$ is here referred to as the effective effort that declines as the water hyacinth level increases. When $0 \leq W_t < \underline{W}$, the effective effort is equal to E_t .

Estimation of (8) enabled us to find the values of the parameters A and D . We have three unknowns: α , r , and K . Any information on r or K will enable us to estimate the values of the parameter α . The estimated $\hat{\alpha}$ is then used to estimate the catchability of fish for the different levels of water hyacinth over years. We are not aware of any reliable estimates of K , the carrying capacity for the aggregate fish stocks in Lake Victoria. However, there have been attempts to

² The derivations behind the two fisheries models are provided in the appendix.

estimate K and the intrinsic rate of growth for Nile perch stocks (Pitcher and Bundy 1995; Pitcher 1995). Estimated values of r or K may be used in the equations containing A and D to find $\hat{\alpha}$.

3.2 A Non-equilibrium Model

In the alternative approach, the fish growth function takes a discrete form. We do not assume that the fisheries are in equilibrium but assume similar effects on catchability.

The growth function is assumed to be (9):

$$B_{t+1} - B_t = rB_t \left(1 - \frac{B_t}{K}\right) - h_t. \quad (9)$$

The harvest and catchability coefficient functions are still assumed to take the forms presented in (2) and (4) above. From (2) above we solve for B_t , the fish stock variable and use the result in (9). A rearrangement yields a dynamic CPUE function below:

$$\frac{h_{t+1}}{\left(1 - \frac{W_{t+1}}{Z}\right)E_{t+1}} = (1+r) \frac{h_t}{\left(1 - \frac{W_t}{Z}\right)E_t} - \frac{r}{\alpha K} \left(\frac{h_t}{\left(1 - \frac{W_t}{Z}\right)E_t} \right)^2 - \alpha h_t, \quad (10)$$

In a backward lag format (10) can be rewritten as:

$$CPUE_t = \beta CPUE_{t-1} - \theta CPUE_{t-1}^2 - \alpha h_{t-1} \quad (11)$$

$$\text{where } CPUE_t = \frac{h_t}{\left(1 - \frac{W_t}{Z}\right)E_t}, \quad \beta = 1+r, \quad \theta = \frac{r}{\alpha K}.$$

Estimation of (11) enables us to capture $\hat{\alpha}$ from the coefficient on h_{t-1} , the catch variable. The intrinsic rate of growth for the lake's stocks is calculated as $(\beta - 1)$ and the carrying capacity, $K = \frac{\beta - 1}{\alpha \theta}$.

Model (8) and (11) are estimated for each country, and lakewide fisheries. The respective catchability coefficients, intrinsic growth rate of stocks, carrying capacity and MSY are estimated. By use of the $\hat{\alpha}$ coefficients and the water hyacinth abundance water hyacinth induced catchability coefficients are calculated.

4. Estimation Issues

In this section, we discuss the econometric issues crucial to the estimation of (8) and (11). The first issue stems from the nature of the data to be used in the analyses. Regressions from time-series data are only useful if series for the variables on the right hand side (RHS) and the left-hand side (LHS) exhibit stationarity. If both the RHS and LHS are non-stationary, regression results will be spurious (Enders 1995).

Non-stationarity of time-series data implies the existence of unit roots in the data. Unit root tests—for example, the augmented Dickey-Fuller (ADF) tests and the Phillip-Perron tests—have been used to establish the order of integration of the series. The distribution theory supporting the ADF tests assumes that the errors are statistically independent and have a constant variance. The Phillip-Perron tests are a generalization of the Dickey-Fuller procedure that allows for fairly mild assumptions concerning the distribution of errors (Enders 1995). We want to caution the researcher performing unit root tests in the presence of structural breaks in the series that it has been observed that when there are structural breaks, the unit root tests above are biased toward the non-rejection of a unit root (see, e.g., Enders 1995; Perron 1989; Doornik and Hendry 1995). Perron (1989) argued that most macroeconomic variables are not characterized by unit root processes. Instead, the variables appear to be trend stationary processes coupled with structural breaks. Perron's test for a unit root in the presence of structural breaks considers a regression equation of the form:

$$\Delta y_t = a_0 + \mu_1 D_L + \mu_2 D_P + a_2 t + a_1 y_{t-1} + \sum_{i=k}^k \beta_i \Delta y_{t-i} + \varepsilon_t, \quad (12)$$

where D_P is a pulse dummy that is equal to 1 in the year of the jump or break, and zero otherwise. D_L is a level dummy that is equal to 1 for all t beginning with the year within which the structural break occurred. Under the presumption of a one-time change in the mean of a unit root process, $a_1 = 1$, $a_2 = 0$, and $\mu_2 = 0$. The alternative hypothesis is that there is a permanent one-time break in the trend stationary process. In this case, $a_1 < 1$ and $\mu_1 = 0$.

The catch and data series that we used in the analysis was generated during periods of instability in the lake (for example, in Uganda, the introduction of alien species in Lake Victoria) and war, among other factors. These have implications for the catches effort and gear composition. Tests for unit roots in the series that took into account the possibilities of structural breaks were therefore used. Furthermore, in estimation of the catch-effort functions, Chow tests for parameter stability during the sample period were conducted to identify structural changes in the data.

The other issue concerns the estimation of (11) where the lagged value of the dependent variable was included in the regressors on the RHS. The correlation between the error term and the lagged dependent variable in (11) made its estimation by OLS regression inappropriate (Greene 1997). Estimation of (11) was thus conducted by two-stage least squares.

5. The Data

This section discusses the sources of the data used in the study. Data on catches and the number of boats engaged in fishing in Lake Victoria in the period 1968–2000 were collected from the fisheries departments of Kenya, Tanzania, and Uganda. There were no detailed surveys conducted on the number of boats in the fisheries for Uganda for the period 1972–89. The recorded figures for the number of fishing boats were estimates provided by the Uganda fisheries authority and were constant for several years. A survey conducted in 1990 showed that there were 8,674 boats in Uganda's Lake Victoria fisheries at the end of 1990, yet the estimate for 1989 was 3,470 boats. The sudden increase in boats between 1989 and 1990 is not plausible and was not supported in interviews with fishermen. It is more likely that boat numbers gradually increased, even though the reported statistics showed constant numbers. We adjusted the data on Uganda's fishing boats with a linear interpolation on the series between 1980 and 1990.³ Similarly, there were no surveys on catches for the Tanzania section during the period 1996–99. Catch data for the period were generated through interpolation.

Due to lack of comprehensive data on other components of fishing effort, we calculated a proxy for effort in the fisheries using the number of boats engaged in the fisheries each year. There is general consensus that nominal fishing effort is measured as a composite of fishing time multiplied by fishing power (Beverton and Holt 1957). Vessel characteristics and gear composition underlie some of the arguments that explain differences in fishing efficiency among fishing units and across different periods of time (Andersen 1999). Due to the complexities involved in measuring fishing effort, the tendency in fisheries modeling has been to choose a measure for effort that biologists believe is related to fishing mortality or, rather, fishing power (Andersen 1999).

³ Moreover, the number of people engaged in fishing was likely to have started increasing in 1980 as result of the end of the war in 1979.

In the literature, several indices of fishing power have been explored. Gulland (1956) and Beverton and Holt (1957) found that fishing power was closely correlated to vessel size, tonnage, and horsepower. While comparing CPUE distribution across different grounds in the Pacific Halibut fishery, Quinn (1985) employed vessel horsepower as the index of effort. Hilborn (1985) standardized effort in terms of fishing time. In his study of estimating growth and stocks of the Indian squid on the southwest coast of India, Mohammed (1996) determined proxies for effort by the number of boat fishing days irrespective of vessel and other gear characteristics. Mkenda (2001) adopted the number of men as the appropriate index of effort for the Zanzibar fishery.

The data at our disposal had no information on vessel size, and we did not have details on how much time was devoted to fishing in the different years. Use of men as an appropriate index of effort would be an option in this case, if comprehensive data on the number of men involved in the fisheries over the period were available. However, there are arguments that, if the number of men in fishing is used as an appropriate index of effort, the differences in skills of skippers should be accounted for (Andersen 1999).

Approximating aggregate annual effort in Lake Victoria fisheries by the number of boats was thus the only alternative. Note also that the available data on boats provided no information on boat characteristics; for example, we could not disaggregate boats according to type, size, or other features. The boats, therefore, were considered homogenous units of effort. The technology for the majority of these vessels, however, is known to be fairly simple and similar. The abundance of the water hyacinth on Lake Victoria over the period studied was estimated as quadratic curves beginning in 1993, maximum growth during 1997, and virtual eradication by the end of 2001 (see, further, Kateregga and Sterner 2007). Variable definitions and descriptive statistics are reported in tables 1 and 2, respectively.

Table 1 Definition of Variables

Variable	Definition
Aggregate catch	Aggregate annual catch in tons
Effort	The number of boats observed in the fisheries each year
CPUE	Catch per unit of effort derived as the average catch per unit of effective effort
Water hyacinth	The abundance of water hyacinth mats (an index based on the area occupied by the weed mats)

Table 2 Descriptive Statistics (1968–2000)

Variable	Mean	Standard deviation	Minimum	Maximum
Kenya				
Aggregate catch	101,339	85,509	14,133	254,040
Effort	5,762	1,939	3,400	10,014
CPUE	20	17	4	53
Water hyacinth	75	1,480	0	4,500
Tanzania				
Aggregate catch	103,386	57,469	40,900	231,547
Effort	5,845	3,296	2,538	15,533
CPUE	19	7.5	8	42
Water hyacinth	277	553	0	1,750
Uganda				
Aggregate catch	64,735	45,813	10,000	146,600
Effort	7,107	4,681	2,643	16,093
CPUE	12	9	4	42
Water hyacinth	685	1,333	0	4,000
Lake-wide				
Aggregate catch	269,460	178,559	73,803	568,043
Effort	18,714	9,530	9,181	41,640
CPUE	15	6	7	27
Water hyacinth	8,837	4,186	0	12,450

For each section of the lake, we constructed the variable effective effort. Lake Victoria is approximately 69,000 km², and is shared by Kenya, Tanzania and Uganda in proportions of 6 percent, 51 percent, and 43 percent, respectively. It has a convoluted shoreline, estimated at 3,450 km. We assumed that the area relevant for fishing was, on average, 3.5 km off the shore, which yielded a lake-wide area for fishing Z that is approximately equal to 12,000 km². The proportions Z_i for each section was determined by each country's share of the lake. We used the water hyacinth abundance and the Z_i s in the formula $\left(1 - \frac{W_t}{Z}\right)E_t$ to transform the number of boats into effective effort used in the estimations in section 6.

6. Estimation

As mentioned earlier, studying the series characteristics is necessary before any estimation can be conducted. Both catch and boat sectional series remained stable between 1968 and 1980, but increased between 1981 and 2000. There were a number of factors that could explain these increments by section. However, the major change common to the three lake sections was the explosion of the Nile perch population after 1980 (Pitcher 1995). Modernization of fishing technology, such as the increased use of outboard engines in the 1980s, the replacement of dug-out canoes by bigger boats, and other changes in the structure of fishing gear might explain the jump in effort series after 1980. Consequently, catch and effort composition before and after 1980 might be different.

We conducted unit root tests on all catch and effective effort series by estimating the equations specified in (12). The parameters for the pulse and level dummies were both zero in all estimations. The Phillip-Perron tests thus failed to detect the structural breaks in the data. This could be a result of the narrowness of the time series used. We instead conducted unit root tests of the augmented Dickey-Fuller format on the data for the sub-samples 1968–82 and 1983–2000. The ADF tests indicated that series for the sub-sample 1983–2000 were stationary. Tests on data from the 1968–1982 sub-sample indicated non-stationarity of the data.

We estimated catch-effort and CPUE functions by section and lake-wide by use of OLS regression. We then examined the stability of the estimated coefficients over the sample period by conducting Chow breakpoint tests. The idea of the Chow test is to fit the equation separately for each sub-sample and to see whether there are significant differences in the estimated equations. A significant difference indicates a structural change in the relationship. The Chow breakpoint test is based on a comparison of the sum of squared residuals obtained by fitting a single equation to the entire sample with the sum of squared residuals obtained when separate equations are fitted to each sub-sample of the data.⁴

⁴ The Chow breakpoint test is an F-test that takes the following form:
$$F = \frac{(u^t u - u_1^t u_1 - u_2^t u_2) / k}{(u_1^t u_1 + u_2^t u_2) / (T - 2k)}$$

where $u^t u$ is the restricted sum of squared residuals, $u_i^t u_i$ is the sum of squared residuals from sub-sample i . T is the total number of observations, and K is the number of parameters in the equation.

The data was divided into two sub-samples, 1968–82 and 1983–2000, and separate functions estimated for each sample. The F-statistics from the 1983 Chow breakpoint were all significant at the 1-percent level. Thus, the hypothesis of the stability of the parameters in the two sub-samples was rejected. Since the data for the earlier period was non-stationary, we based the analysis on the data for 1983–2000.

7. Results

This section has the results for all of Lake Victoria and national functions for the sub-sample period 1983–2000. The results from the static analysis are reported in subsection 7.1, and the dynamic analysis in 7.2. The effect of water hyacinths is discussed in 7.3.

7.1 The Catch-Effort Functions

The results from the static analysis are reported in table 3. Coefficients in the lake-wide, Tanzania, and Uganda estimations had the expected signs and were all significant at the 1-percent level. The estimated coefficients for the Kenyan section had the expected signs; however, the coefficient on the effort-squared variable was not significant. The DW statistics were less than 1 for all sections indicating first order autocorrelation. Autocorrelation has been a common problem in fisheries model estimations (Hannesson 1983). It reflects the fact that catches in any period depend on catches in earlier periods.

The calculations for the solution of the values for $\hat{\alpha}$, r , and K required us to have some prior information on one of the three unknowns. As noted above, no estimates of K for aggregate stocks were available for Lake Victoria. However, Pitcher and Bundy (1995) conducted an assessment of the lake-wide Nile perch stocks using catch and effort data 1979–90. Their results showed a population growth rate for Nile perch between 1.06 and 1.61. This estimated range for the intrinsic growth rates seemed to be rather high, but their justification was that it was quite reasonable for the perch species. The carrying capacity for these species was estimated to be between 1,000 and 1,870 kilotons. From these estimates, the implied range for the maximum sustainable yield for Nile perch was between 279 and 489 kilotons per year.

We used the lower value of the estimated growth rate for Nile perch from the Pitcher and Bundy study to solve for the values of α and K from the estimated parameters in the static functions. We assumed a common population growth rate for the Nile perch lake-wide. We did not know the growth rates for the other species, but since tilapia and dagaa (a general name in Tanzania for various types of sardine-like fish) only constitute small proportions of catch from

Table 3 Estimated Carrying Capacity: Static Analysis (1983–2000)

Variable	Lake section			
	Lake-wide	Kenya	Tanzania	Uganda
	Coefficient	Coefficient	Coefficient	Coefficient
Effort	33.3 *** (10.2)	46.7** (3.4)	35.1*** (9.92)	26.6*** (8.1)
Effort ²	-0.0006*** (-5.11)	-0.003 (-1.19)	-0.002 *** (-4.92)	-0.0014*** (-5.32)
Adjusted R ²	0.41	0.90	0.93	0.38
Sigma	87,500	57,300	42,300	27,400
RSS	1.22E+11	5.26E+10	2.86E+10	1.20E+10
Log likelihood	-229.3	-221.7	-216.2	-208.4
DW	0.46	0.28	0.97	0.96
Observations	18	18	18	18
α	0.000019	0.000063	0.000054	0.000054
K (metric tons)	1,752,000	741,000	649,000	492,000
MSY [‡] (metric tons)	464,000	196,000	172,000	130,000

Notes: The value of r ($r=1.06$) used in these calculations was derived from Pitcher and Bundy (1995).

***, **, * indicate significance at 1%, 5%, and 10%, respectively.

The figures in parentheses are t-statistics.

[‡] MSY is maximum sustainable yield.

the lake, we used these figures for the total stock. The estimated catchability coefficients, $\hat{\alpha}$ and the carrying capacity K and maximum sustainable yield for Lake Victoria, and for the three sections of the lake, are reported in table 3. The estimated carrying capacity for the lake was 1,752 kilotons, yielding a maximum sustainable yield of 464 kilotons. From the results, the catchability of fish was highest in the Kenyan section.

7.2 Dynamic CPUE Functions

In this section, we estimate and discuss the results from the dynamic lake-wide CPUE function. The estimation was carried out by two-stage least squares, using the second period lags as instruments. The results from the dynamic CPUE functions are given in table 4.

Table 4 Estimated Carrying Capacity: Dynamic Analysis (1983–2000)

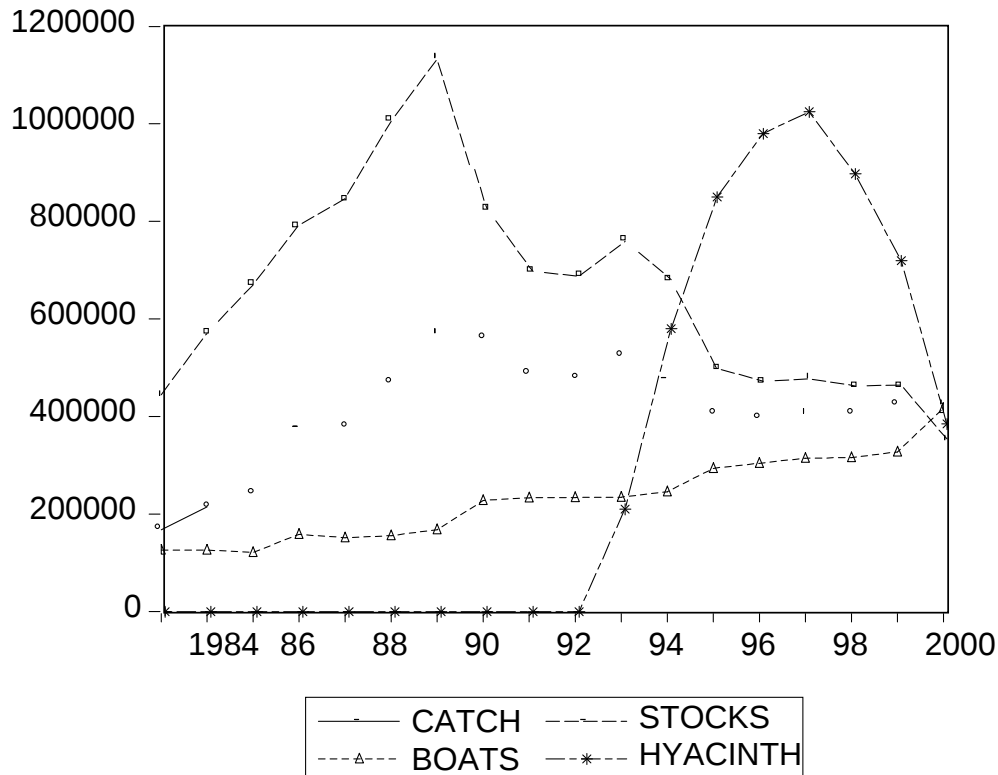
Variable	Coefficient	Estimated parameters	Value
$CPUE_{t-1}$	2.02*** (6.20)	R	1.02
$CPUE_{t-1}^2$	-0.021* (-2.55)	α	0.00003
$Catch_{t-1}$	-0.00003** (-3.35)	K (metric tons)	1,619,000
Adjusted R^2	0.83	$MSY^{\dagger}$ (metric tons)	413,000
Sigma	2.71		
RSS	95.29		
DW	1.84		

The dynamic results indicated that the lake-wide population growth rate was 1.02 and the carrying capacity for the whole lake was 1,619 kilotons. The results for K from both models were fairly close. The estimated lake-wide stocks, water hyacinth coverage, catch, and number of boats are plotted in figure 1.⁵ The figure shows that fish stocks appear to have risen from 440 kilotons in 1983 to a maximum of 1,130 kilotons in 1989. The number of boats increased during this period, and catch increased until 1989, but then declined. The increase in water hyacinth biomass between 1992 and 1997 appears to have masked how the increase in number of boats kept effective effort from increasing.

However, despite the hyacinth infestation, fish stocks declined. In the period 1998–2000, where the amount of water hyacinth mats were reduced on the lake, effective effort increased and

⁵ The estimated stocks are $B_t = \frac{h_t}{\alpha \left(1 - \frac{W_t}{Z}\right) E_t}$, with $\alpha = 0.00003$ as estimated in the dynamic function.

Figure 1 Estimated Lake-Wide Fish Stocks, Catch, Number of Boats, and Water Hyacinth Mats, 1983–2000



Estimated stocks and catch are measured in metric tons. The number of boats is scaled up by a factor of 10, and the hectares of hyacinth mats scaled up by 100.

even sustained a temporary increase in catch. But, this led to an even faster fall in fish stocks, a sign that the fisheries were in disequilibrium. This pointed strongly to a possible collapse of the fisheries. Catches from the lake consistently declined in the period 2001–2003 and a drastic reduction in fish stocks has been acknowledged by the Uganda fisheries department.

7.3 Water Hyacinth Induced Catchability Coefficients

The estimated values of $\hat{\alpha}$ and the hectares covered by water hyacinths were incorporated in catchability coefficients function in (4) in order to calculate the water hyacinth-

induced catchability coefficients.⁶ These were calculated for the years 1993–2000. Both the results from the static and dynamic analyses revealed that, on average, the catchability of fish was reduced by 45 percent, 2 percent, and 6 percent in the Kenya, Tanzania, and Uganda sections of Lake Victoria, respectively.⁷ The larger reduction in the catchability of fish in Kenya's section may be explained by the high abundance of water hyacinth mats in this area, compared, for example, to Tanzania. These results revealed that the presence of water hyacinth in the lake drastically reduced the catchability of fish in the Kenyan section of Lake Victoria. The effects on the Tanzania and Uganda sections were milder.

We only had access to some partial catch data for the latest couple of years for the Uganda section of the lake. According to the Uganda Statistical Abstract 2006, catches of Nile perch continued to increase until 2004, but declined severely (by 20 percent) in 2005. Newspaper reports in 2007 suggested that the decline is becoming even more serious (*New Vision* [Kampala] 2007). This fits well with the analysis here that the hyacinths somewhat delayed overfishing and that, now they are gone, overfishing has accelerated again.

8. Conclusions

We explored the effect of the water hyacinth on the catchability of fish in the Kenya, Tanzania, and Uganda fisheries of Lake Victoria by incorporating the water hyacinth biomass as a negative factor in the catchability coefficient. The results indicate that the catchability of fish in the Lake Victoria fisheries was reduced by a factor of 2–45 percent during the period when the lake was highly infested by the water hyacinth. The decline was greatest in the Kenya section. The weed also caused numerous problems to shipping, hydropower, and other activities besides fishing. It is therefore understandable that strong measures were pursued to control its growth, but what caused the excess supply of nutrients and sparked the flourishing growth of the water hyacinth is still unknown. It should also be recognized that although the water hyacinths were a serious problem, they ironically also had the benefit of (temporarily) reducing fishing pressure.

⁶ Water hyacinth-induced catchability coefficients were calculated by inserting the estimated catchability coefficients lake-wide and for each county's fisheries, and the period t water hyacinth biomass calculated in the

formula: $q(W_t) = \alpha \left(1 - \frac{W_t}{Z} \right)$.

⁷ The percentage reductions in the catchability of fish were calculated by making a comparison between the water hyacinth-induced catchability coefficients and the unadjusted coefficients in tables 3 and 4.

At present, there appears to be a drastic decline in fish stocks in Lake Victoria. The need to substantially reduce effort on the lake and take other measures to salvage the stocks from the path of extinction is raised by this study. It will become even more crucial when the hyacinths are cleared and this obstacle to fishing is removed. This may well be the explanation for the current crisis.

The sample period considered in the estimated results is relatively short, due to the structural changes in the data that emerged from changes in catch and effort composition after 1980. Better estimates of the fisheries parameters may be calculated in the future with longer time series. Another area for future research would be to disaggregate the data by species and to model not only the effect on catchability but also the biological effect of hyacinths on the fish stock function.

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Appendix

Derivations of the Static and Dynamic CPUE Functions

A.1 Static CPUE Function

Assumptions:

$$h_t = q(W_t)E_t B_t \quad (A1)$$

$$q(W_t) = \left(1 - \frac{W_t}{Z}\right) \quad (A2)$$

$$\frac{dB}{dt} = rB_t \left(1 - \frac{B_t}{K}\right) - h_t \quad (A3)$$

Substitution for q in (1) yields:

$$h_t = \left(1 - \frac{W_t}{Z}\right) E_t B_t . \quad (A4)$$

We used (A4) to substitute for h_t in the growth function in (A3) and used the assumption of equilibrium in the fishery. That is, the fishery is in a state in which catch always equals the growth in fish stocks. This implies that:

$$rB_t \left(1 - \frac{B_t}{K}\right) = \left(1 - \frac{W_t}{Z}\right) E_t B_t . \quad (A5)$$

Solution for B_t in (A4) yields:

$$B_t = \left(K - \frac{\alpha K}{r} \left(1 - \frac{W_t}{Z}\right) E_t \right) . \quad (A6)$$

Substitution for B_t in (A1) yields the catch-effort function as:

$$h_t = \left(K - \frac{\alpha K}{r} \left(1 - \frac{W_t}{Z}\right) E_t \right) * \alpha \left(1 - \frac{W_t}{Z}\right) E_t = \alpha K \left(1 - \frac{W_t}{Z}\right) E_t - \frac{\alpha^2 K}{r} \left(1 - \frac{W_t}{Z}\right)^2 E_t^2 , \quad (A7)$$

$$A = \alpha K, \quad D = \frac{\alpha^2 K}{r} .$$

A.2 Dynamic CPUE Function

Assumptions:

$$h_t = q(W_t)E_t B_t \quad (\text{A9})$$

$$q(W_t) = \alpha \left(1 - \frac{W_t}{Z} \right) \quad (\text{A10})$$

$$B_{t+1} - B_t = \dot{B} = rB_t \left(1 - \frac{B_t}{K} \right) - h_t \quad (\text{A11})$$

Substitution for q in (A9) yields:

$$h_t = \alpha \left(1 - \frac{W_t}{Z} \right) E_t B_t . \quad (\text{A12})$$

The differences in the two fisheries models emerge from, first, the form of the growth function assumed. For the static CPUE function, the fish stock growth function is continuous; in the dynamic analysis, the fish stock growth function is assumed to take a discrete form. Further, the assumption of the steady state existence is not necessary here. We use (A12) to solve for B_t :

$$B_t = \frac{h_t}{\alpha \left(1 - \frac{W_t}{Z} \right) E_t} \quad (\text{A13})$$

Substitution for B_t in the fish stock growth function in (A11) yields:

$$\frac{h_{t+1}}{\alpha \left(1 - \frac{W_{t+1}}{Z} \right) E_{t+1}} - \frac{h_t}{\alpha \left(1 - \frac{W_t}{Z} \right) E_t} = r \frac{h_t}{\alpha \left(1 - \frac{W_t}{Z} \right) E_t} - \frac{r}{K} \left(\frac{h_t}{\alpha \left(1 - \frac{W_t}{Z} \right) E_t} \right)^2 - h_t . \quad (\text{A14})$$

Multiplying both sides of (A14) by α , writing $\frac{h_t}{\left(1 - \frac{W_t}{Z} \right) E_t}$ as CPUE_t , a rearrangement

and the use of a backward lag operator yields:

$$\text{CPUE}_t = \beta \text{CPUE}_{t-1} - \theta \text{CPUE}_{t-1}^2 - \alpha h_{t-1} , \quad (\text{A15})$$

where $\beta = 1 + r$, $\theta = \frac{r}{\alpha K}$.