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A Dynamic Enforcement Strategy to Improve Compliance with Environmental Regulations

Xiao-Bing Zhang and Jing Xu

Environment for Development Centers

Central America
Research Program in Economics and Environment for Development in Central America
Tropical Agricultural Research and Higher Education Center (CATIE)
Email: centralamerica@efdinitiative.org

Chile
Research Nucleus on Environmental and Natural Resource Economics (NENRE)
Universidad de Concepción
Email: chile@efdinitiative.org

China
Environmental Economics Program in China (EEPC)
Peking University
Email: china@efdinitiative.org

Ethiopia
Environment and Climate Research Center (ECRC)
Ethiopian Development Research Institute (EDRI)
Email: ecrc@edri.org.et

Kenya
School of Economics
University of Nairobi
Email: kenya@efdinitiative.org

South Africa
Environmental Economics Policy Research Unit (EPRU)
University of Cape Town
Email: southafrica@efdinitiative.org

Sweden
Environmental Economics Unit
University of Gothenburg

Email: info@efdinitiative.org

Tanzania
Environment for Development Tanzania
University of Dar es Salaam
Email: tanzania@efdinitiative.org

USA (Washington, DC)
Resources for the Future (RFF)
Email: usa@efdintiative.org

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A Dynamic Enforcement Strategy to Improve Compliance with Environmental Regulations

Xiao-Bing Zhang and Jing Xu

Abstract

This paper develops a stochastic dynamic programming model to investigate a type of dynamic enforcement strategy where the penalties for violations of environmental regulations are based on not only the current level of violations but also the firms' past noncompliance records. The results show that firms' optimal level of noncompliance would be a decreasing function of their accumulated noncompliance record and that a more stringent enforcement strategy can reduce the expected fines due to the reduced violations. Comparisons with the repeated static enforcement strategy indicate that the dynamic enforcement strategy can be superior in terms of reducing both violations and enforcement efforts.

Key Words: dynamic enforcement, stochastic dynamic programming, noncompliance

JEL Codes: C61, C63, Q58

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1. Introduction

Firms' compliance with environmental regulations depends on effective monitoring and enforcement by the regulator. Usually, monitoring refers to the efforts of the regulator to find out what the firms are doing and how it compares with what they are required to be doing, while enforcement refers to the actions taken on the basis of monitoring evidence to penalize undesired behavior (Russell 2001). On one hand, the absence of monitoring causes the regulator to lack the evidence necessary to implement enforcement. On the other hand, without enforcement, there is no deterrence for the violating firms and no incentive for them to correct their behavior. Therefore, enforcement is especially important to induce firms to comply with regulations, and it always shows up in the literature as a penalty or penalty function (Russell 2001).

Most studies on incomplete compliance with environmental regulations are based on a static framework of enforcement where the penalties depend only on the firm's current level of violation, i.e., violations by a firm in the past do not affect the penalties for its current violation (see, e.g., Harford 1978; Garvie and Keeler 1994; Heyes 1998; Sandmo 2002; Macho-Stadler and Pérez-Castrillo 2006; Villegas-Palacio and Coria 2010; Arguedas et al. 2010). However, in reality, the regulator can keep a record of a firm's past compliance/violations and the extent to which the firm violates the regulations. The recorded information on past violations can affect the penalties that the firm faces if it is caught committing another violation. Most commonly, a firm with a "bad" compliance history faces a larger penalty for the same extent of violations, compared with the firm with a "good" compliance record. For instance, the monetary penalties for violating the Air Pollution Control Regulation in Beijing are doubled for firms that are caught violating for the second time. Similarly, in the United States, monetary penalties aimed at correcting environmental violations depend on three factors:

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* Xiao-Bing Zhang, School of Economics, Renmin University of China, Beijing, xbzhmail@gmail.com. Jing Xu, School of Public Finance and Taxation, Southwestern University of Finance and Economics, Chengdu, China. Email: xuj@swufe.edu.cn. Financial support from the Swedish International Development and Cooperation Agency through the Environment for Development Initiative is gratefully acknowledged.

the severity and magnitude of the violation, the violation pattern or record, and the actions taken by the violator to correct the problem. Empirically, Oljaca et al. (1998) estimate the penalty function for water quality violations and find that historical compliance records strongly influence penalty levels. Rousseau (2009) and Rousseau and Blondiau (2014) present similar conclusions, showing that, in practice, fines are higher for repeat offenders. Moreover, Blondiau and Rousseau (2011) theoretically and empirically confirm that repeat offenders are given tougher penalties, while intentionality does not have a direct effect on punishment; this indicates that repeat violation is a more objective punishment criterion, easier to observe compared to state of mind.

Some previous studies have highlighted the importance of past compliance records in the design of enforcement strategy. Harrington (1988) presented a targeted enforcement strategy in which the regulated firms are treated differently, with the monitoring and enforcement resources concentrated on the firms in the target group. Compliance history plays a key role in classifying firms into a target or non-target group. Specifically, a firm in the non-target group would be moved to the target group once caught violating the environmental regulation in the previous period; conversely, a firm found complying in the past could have the chance to escape from the target group. Harrington's paper triggered a wave of studies concerning the dynamic aspects of regulatory enforcement, though with different focuses, for instance, on the robustness of Harrington's results with generalized measures of social cost (Harford 1991; Harford and Harrington 1991), on the extension of Harrington's model under asymmetric information on compliance cost distribution (Raymond 1999), and on an extension with improved transition structure for the two-group audit framework (Friesen 2003; Coria and Zhang 2015).

In these studies, compliance/violation history is critical for formulating enforcement strategies. However, past compliance goes into Harrington's model in a way that does not accumulate over time. That is, the transition of firms between groups and the penalties for violations depend on the most recent record of compliance only. A constant violator receives the same penalty as a firm that violated only in the last period. In reality, the mechanism is more flexible. The regulator is likely to make use of a firm's accumulated compliance records going back many years rather than just the most recent record to determine the penalty for its current violation. Hence, the regulated firms would behave strategically in the dynamic environment because their current compliance decisions can have an effect on future penalties. Moreover, because the monitoring is random and a firm's compliance status in one period can only be known if this firm is actually inspected in that period, the regulated firms are facing a stochastic environment in their compliance/violation decision-making.

In this paper, we study the effect of a dynamic enforcement strategy where the compliance history is taken into account to penalize violations. The enforcement is "dynamic" in the sense that the penalty is a function of the accumulated compliance record which evolves over time and that the regulated firms look forward to make their current compliance decisions. Therefore, we refer to such an enforcement strategy as *dynamic enforcement* hereinafter. To investigate firms' behavior under such a dynamic enforcement strategy, this paper develops a model based on stochastic dynamic programming. This study finds that firms' optimal level of noncompliance is a decreasing function of their accumulated noncompliance and that a more stringent enforcement strategy can reduce the expected fines due to the reduced violations. When the noncompliance record is irreversible, firms will always comply in the long run. With proper design of penalty parameters, the expected monetary penalty in the dynamic enforcement strategy can be lower than that in the static enforcement strategy in the long run, which implies that, under some circumstances, the dynamic enforcement strategy can improve firms' compliance with environmental regulations and can meanwhile require a lower enforcement effort, compared with the traditional static enforcement strategy discussed in the literature.

The rest of the paper is organized as follows. Section 2 develops the model for the static enforcement strategy. The theoretical analysis of a dynamic enforcement strategy is presented in Section 3. Section 4 compares firms' behavior in the dynamic enforcement strategy with that in the static enforcement strategy. In Section 5, the theoretical predictions are illustrated with numerical simulations. Finally, Section 6 concludes.

2. The Static Enforcement Model

Before going through the model for the dynamic enforcement strategy, we first present the static enforcement model as a benchmark. "Static" here means that the penalty for a violating firm depends only on its current level of violation and that its compliance history does not matter. In this case, the regulated firms behave non-strategically and solve a sequence of static optimization problems. In each period t, the firm decides to comply with or violate the emission standard $\bar{q}$ by choosing the level/extent of violation v_t ($v_t = 0$ represents compliance and $v_t > 0$ represents violating the standard). The emissions in period t therefore would be: $q_t = \bar{q} + v_t$. For any final emission level q, let the abatement cost function of the firm be denoted as $C(q)$, which is strictly convex and decreasing in the level of emissions q, i.e., we assume a quadratic form for the abatement cost: $C(q)=\frac{a}{2}q^2-bq+f$ (see, e.g., Hoel and Karp 2002), with $C'(q)=aq-b<0$ and $C''(q)=a>0$. The monitoring probability for a firm is π. Once the firm is found violating in the amount v, the penalty would be $\Phi(v)=\frac{\phi}{2}v^2$, where ϕ represents the marginal penalty rate. Therefore, under the static enforcement strategy, a representative firm solves the following problem in period t:

$$\min_{\{v_t\}} C(\bar{q}+v_t)+\pi\cdot\Phi(v_t) \qquad s.t.\, v_t \geq 0 \tag{1}$$

The Lagrangian would be:

$L=C(\bar{q}+v_t)+\pi\cdot\Phi(v_t)-\omega\cdot v_t$ where ω is the Lagrange multiplier.

Kuhn-Tucker conditions yield:

$$\frac{\partial L}{\partial v_t}=C'(\bar{q}+v_t)+\pi\cdot\Phi'(v_t)-\omega=0,\ \omega\geq 0 \text{ with } \omega=0 \text{ if } v_t>0 \tag{2}$$

For compliance $v_t=0$, we have $\omega\geq 0$. That is, $C'(\bar{q})+\pi\cdot\Phi'(0)\geq 0$, implying that, in order to induce compliance, the inspection probability should be higher than a threshold $\pi\geq\pi^*$, where $\pi^*=-\frac{C'(\bar{q})}{\Phi'(0)}>0$. If $|C'(\bar{q})|$ is high, we can have: $\pi^*>1$. In other words, even though the regulators use the most intensive monitoring, the firms still violate because their marginal abatement cost at $q=\bar{q}$ is too high. If $C'(\bar{q})=0$, we have: $\pi^*=0$. That is, even with a very small monitoring probability, firms will always comply.

If the monitoring probability is lower than the critical probability π^*, i.e., $\pi<\pi^*$, firms will commit violations. For noncompliance, $v_t>0$, we have $\omega=0$. Condition (2) becomes the first order condition for an interior solution of (1):

$$C'(\bar{q}+v_t)+\pi\cdot\Phi'(v_t)=0 \tag{3}$$

The optimal level of violation v_t^* is determined by (3), i.e.,

$$-C'(\bar{q}+v_t^*)=\pi\cdot\Phi'(v_t^*) \tag{4}$$

which equates the marginal abatement cost (marginal saving) and the expected marginal penalty.

Plugging in the corresponding function forms, condition (4) becomes $a[\overline{q}+v_t^*]-b+\pi[\phi v_t^*]=0$, from which we have:

$$v_t^* = \frac{b-a\overline{q}-\pi\phi}{a} \qquad (5)$$

The optimal level of violation is independent of the fixed cost f and is decreasing in the monitoring probability π and the penalty rate ϕ: the more frequent the inspection or the harsher the penalty, the lower the extent of violations.

Because firms do not look forward to make their compliance decisions (i.e., they behave non-strategically) under the static enforcement strategy, the same amount of violation is chosen in each time period, that is, $v_{t+1}^* = v_t^*$ for all t. Therefore, if the monitoring probability is higher than the critical probability, i.e., if $\pi \geq \pi^*$, the firm chooses to comply in all time periods (i.e., $v_t = 0$, $t = 1,2,\ldots$); otherwise, if $\pi < \pi^*$, the firm will always choose the same level of violation $v_t^* = \frac{b-a\overline{q}-\pi\phi}{a}$ for each time period. Because the case of incomplete compliance is more interesting, the analysis will be focused on the situation where the monitoring is not frequent enough to support full compliance, i.e., $\pi < \pi^*$. This is consistent with the reality that the budget of regulators is limited and, for most sources of pollution, the surveillance frequency is quite low (Harrington, 1988).

3. The Dynamic Enforcement Model

3.1 A Dynamic Enforcement Model Based on Noncompliance Record

Under the dynamic enforcement strategy, a representative firm decides to comply with or violate the emission standard $\overline{q}$ in each time period t by choosing the level of violation $v(t)$ ($v(t)=0$ for compliance, and $v(t)>0$ for violation).[1] Therefore, the

[1] To differentiate the dynamic model from the static model above, where we denote the violation in period t as v_t, here we use $v(t)$ instead to denote the level of violation in period t.

abatement cost associated with the violation $v(t)$ can be written as:

$C[q(t)] = \frac{a}{2}[q(t)]^2 - b[q(t)] + f$, where $q(t) = \bar{q} + v(t)$.

The regulatory agency keeps a record of the firm's compliance history. A firm's compliance status in one period can only be known if it receives an inspection in that period. Otherwise, the regulator will lack the evidence to add violations into the noncompliance record. Hence, the evolution over time of a firm's noncompliance record follows:

$$S(t+1) = \begin{cases} \Delta \cdot S(t) + v(t) & \text{if inspected in period } t \\ \Delta \cdot S(t) & \text{if not inspected in period } t \end{cases} \quad \text{with } S(0) = S^0 = 0 \qquad (6)$$

where $S(t)$ is the cumulative violation of a representative firm which has been recorded by the regulator at the beginning of period t. $\Delta = 1 - \rho$ $(0 \le \Delta \le 1)$ indicates the decay in the record that the regulatory agency keeps for a firm and ρ can be considered as the rate of decay. $\rho = 0$ (i.e., $\Delta = 1$) would represent the case of an irreversible cumulative compliance record, which implies that the regulator maintains the compliance records indefinitely. As can be seen from (6), a firm's violation in a certain time period will be recorded and accumulated by the regulator only if that firm is actually inspected in that period. Otherwise, the regulator will lack the evidence to add violations into the noncompliance record. Given that whether a firm will be audited in a certain period is random and depends on the monitoring probability, the dynamics of the violation record (6) are essentially stochastic.

In the case of being inspected and found in violation, the firm has to pay a penalty that consists of two parts: one part is based on the firm's current violation: $\Psi(v(t)) = \frac{\varphi}{2}[v(t)]^2$, and the other is associated with the firm's past violation record:

$$\Omega(S(t)) = \frac{\gamma}{2}[S(t)]^2 \qquad (7)$$

where γ reflects the magnitude of the penalty on cumulative violations. Therefore, under the dynamic enforcement strategy, the total penalty for a violating firm in period t would be: $\Lambda(S(t),v(t)) = \Psi(v(t)) + \Omega(S(t))$.[2]

Recall that, under the static enforcement strategy, the penalty for a violating firm is set regardless of its past compliance record, with $\Phi(v) = \frac{\phi}{2}v^2$. Therefore, if we set $\varphi = \phi$ and $\gamma = 0$, the penalties under both the static and the dynamic enforcement strategy coincide for the same extent of detected violation, which is also the case if $\varphi = \phi$ and $\Delta = 0$.

Being aware that its violation behavior today may lead to future consequences of a higher penalty, which depends on the accumulated violations under the dynamic enforcement strategy, a firm looks forward to make its compliance decisions. Specifically, a representative firm chooses the optimal level of violation to solve the following stochastic dynamic programming problem:

$$M(S(t)) = \min_{v(t)\geq 0}\left\{C(\bar{q}+v(t)) + \pi\cdot[\Psi(v(t)) + \Omega(S(t))] + \delta\mathbb{E}\{M(S(t+1))\}\right\} \tag{8.1}$$

subject to the dynamics of the state variable S as described by (6). $M(S)$ is the minimized value function and the expectation sign $\mathbb{E}\{\cdot\}$ in (8.1) is due to the uncertainty embodied in $S(t+1)$ (see (6)). Recursively, (8.1) can be rewritten as (time notation is ignored):

$$M(S) = \min_{v\geq 0}\left\{C(\bar{q}+v) + \pi\cdot[\Psi(v) + \Omega(S)] + \delta[\pi\cdot M(\Delta S + v) + (1-\pi)M(\Delta S)]\right\} \tag{8.2}$$

That is,

$$M(S) = \min_{v\geq 0}\left\{C(\bar{q}+v) + \pi\cdot[\Psi(v) + \Omega(S) + \delta M(\Delta S + v)] + (1-\pi)\delta M(\Delta S)\right\}. \tag{8.3}$$

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2 In the case that $v(t)=0$, there is no current violation, but the firm may still have to pay a fine according to its past violations (if any) once inspected, which is consistent with the specification in Hentschel and Randall (2000).

With probability π, the representative firm is inspected and is required to pay the penalty for its violation (if any) in that period. Moreover, the violation will be accumulated in the record that the regulator keeps. With probability $1-\pi$, the firm is not inspected, resulting in zero penalty and no increment in its violation record.

The first order condition for interior solution of (8.3) yields:

$$-C'(\bar{q}+v)=\pi\cdot[\Psi'(v)+\delta M'(\Delta S+v)] \tag{9}$$

which says that the marginal saving in abatement by emitting (violating) one more unit in the current period should be equal to the sum of the expected marginal penalty in the current period and the expected marginal costs of increasing the state variable (i.e., the cumulative noncompliance record) in the next period. That is, under the dynamic enforcement strategy, firms would take into account the effect of their current behavior on the expected cost in their future decision-making process, which is absent in firms' decisions under the static enforcement strategy, as indicated in (4).

Due to the linear-quadratic structure of the model, we know the value function will be in a quadratic form:

$$M(S)=m_0+m_1S+\frac{1}{2}m_2S^2 \tag{10}$$

where m_0, m_1, and m_2 are the coefficients to be determined. Plugging in the functional forms of the abatement cost and penalties together with (10) into (9), we have:

$$a[\bar{q}+v]-b+\pi[\varphi v]+\delta\pi[m_2(\Delta S+v)+m_1]=0\,. \tag{11}$$

Accordingly, the optimal violation level is:

$$v^*=\frac{b-a\bar{q}-\delta\pi m_1}{a+\pi(\varphi+\delta m_2)}-\frac{\delta\pi m_2\Delta}{a+\pi(\varphi+\delta m_2)}S=K_0-K_1S \tag{12.1}$$

where we have:

$$K_0=\frac{b-a\bar{q}-\delta\pi m_1}{a+\pi(\varphi+\delta m_2)} \tag{12.2}$$

$$K_1 = \frac{\delta\pi m_2\Delta}{a+\pi(\varphi+\delta m_2)} \tag{12.3}$$

By plugging the optimal violation strategy (12.1) into the Bellman equation (8.2) or (8.3) and eliminating the maximization, we have

$$\begin{aligned} m_0 + m_1 S + \frac{1}{2}m_2 S^2 &= \frac{a}{2}[\bar{q}+K_0-K_1S]^2 - b[\bar{q}+K_0-K_1S] + c_0 + \pi\cdot[\frac{\varphi}{2}(K_0-K_1S)^2 + \frac{\gamma}{2}S^2] \\ &+ \delta\left[m_0 + m_1\Delta S + \frac{1}{2}m_2\Delta^2 S^2 + \pi m_1(K_0-K_1S) + \pi m_2\Delta S(K_0-K_1S) + \frac{1}{2}\pi m_2(K_0-K_1S)^2\right] \end{aligned} \tag{13}$$

Then, equating the coefficients of S^2, S, and 1 on both sides of (13) yields:

$$\frac{1}{2}m_2 = \frac{a}{2}(K_1)^2 + \pi[\frac{\varphi}{2}(K_1)^2 + \frac{\gamma}{2}] + \delta[\frac{1}{2}m_2\Delta^2 - \pi m_2\Delta K_1 + \frac{1}{2}\pi m_2(K_1)^2]$$

(14.1)

$$m_1 = -a(\bar{q}+K_0)K_1 + bK_1 - \pi\varphi K_1K_0 + \delta[m_1\Delta - \pi m_1K_1 + \pi m_2\Delta K_0 - \pi m_2K_0K_1] \tag{14.2}$$

$$m_0 = \frac{a}{2}(\bar{q}+K_0)^2 - b(\bar{q}+K_0) + c_0 + \pi\frac{\varphi}{2}(K_0)^2 + \delta[m_0 + \pi m_1K_0 + \frac{1}{2}\pi m_2(K_0)^2] \tag{14.3}$$

After the substitution of K_1 in (12.3) into (14.1) and some re-arrangement, we have:

$$\pi\delta[1-\delta\Delta^2(1-\pi)][m_2]^2 + [(1-\delta\Delta^2)(a+\pi\varphi) - \pi^2\gamma\delta]m_2 - \pi\gamma(a+\pi\varphi) = 0 \tag{15}$$

Choosing the unique positive root to ensure the convexity of the value function, m_2 solves as:

$$m_2 = \frac{[\pi^2\gamma\delta - (1-\delta\Delta^2)(a+\pi\varphi)] + \sqrt{[(1-\delta\Delta^2)(a+\pi\varphi) - \pi^2\gamma\delta]^2 + 4\pi\delta[1-\delta\Delta^2(1-\pi)]\cdot\pi\gamma(a+\pi\varphi)}}{2\pi\delta[1-\delta\Delta^2(1-\pi)]} \tag{16.1}$$

Besides, plugging (12.2)-(12.3) into (14.2)-(14.3), after some re-arrangement, we have:

$$m_1 = \frac{\pi m_2\delta\Delta[b-a\bar{q}]}{(1-\delta\Delta)(a+\pi\varphi) + \pi\delta m_2[1-\delta\Delta(1-\pi)]} \tag{16.2}$$

$$m_0 = -\frac{1}{2(1-\delta)}\frac{[b-a\overline{q}-\delta\pi m_1]^2}{a+\pi(\varphi+\delta m_2)} + \frac{1}{1-\delta}[\frac{a}{2}\overline{q}^2 - b\overline{q} + c_0] \tag{16.3}$$

Therefore, we have obtained the coefficients for the value function $M(S)$ as in (16.1)-(16.3). Combining the information in (16.1)-(16.3) and (12.1)-(12.3), the optimal level of violation is characterized by a function of the state variable and model parameters. Again, similar to the argument for the static enforcement strategy (see Section 2), the analysis under the dynamic enforcement strategy will also be focused on the situation where the monitoring is low, as in reality.

3.2 Firms' Behavior under the Dynamic Enforcement Strategy

Having characterized the optimal violation level in the analytical model, we investigate firms' behavior patterns under the dynamic enforcement strategy. Based on the calculations that $m_2 > 0$ and $m_1 > 0$ (because $b - a\overline{q} > 0$), we know from (12.3) that $K_1 > 0$. Hence, firms' level of violation decreases with their cumulative noncompliance record, which gives the first proposition below. That is, a firm with a worse compliance record tends to violate less due to the deterrent effect of the cumulative violation record. Specifically, once caught violating, a firm with higher cumulative violation will have to pay a larger penalty.

Proposition 1. *The optimal violation level for a firm is a decreasing function of its cumulative noncompliance record.*

Because the optimal level of violation decreases as the cumulative violation becomes larger, there exists a critical value for the noncompliance record at which no further violations occur, that is, the optimal level of current violation would be zero when the cumulative violation record reaches such a critical level.

Recall the optimal violation from (12.1),

$$v = \frac{b-a\overline{q}-\delta\pi m_1}{a+\pi(\varphi+\delta m_2)} - \frac{\delta\pi m_2\Delta}{a+\pi(\varphi+\delta m_2)}S = K_0 - K_1 S$$

We know that the violation converges to zero as the cumulative noncompliance record approaches the critical value:

$$\tilde{S} = \frac{K_0}{K_1} = \frac{b-a\overline{q}-\delta\pi m_1}{\delta\pi m_2\Delta} \tag{17}$$

Taking the derivatives of m_2 with respect to parameter γ yields:

$$\frac{\partial m_2}{\partial \gamma} = \left[\pi^2\delta + \frac{2\pi^2\delta[1-\delta\Delta^2(1-\pi)][a+\pi\varphi] - \pi^2\delta(1-\delta\Delta^2)(a+\pi\varphi) + \pi^2\delta\pi^2\gamma\delta}{\sqrt{[(1-\delta\Delta^2)(a+\pi\varphi) - \pi^2\gamma\delta]^2 + 4\pi^2\delta\gamma[1-\delta\Delta^2(1-\pi)](a+\pi\varphi)}} \right] \Big/ 2\pi\delta[1-\delta\Delta^2(1-\pi)]$$

Because $1-\delta\Delta^2(1-\pi) > 1-\delta\Delta^2$, we have $2\pi^2\delta[1-\delta\Delta^2(1-\pi)](a+\pi\varphi) - \pi^2\delta(1-\delta\Delta^2)(a+\pi\varphi) > 0$, which implies $\frac{\partial m_2}{\partial \gamma} > 0$.

Similarly, for m_1, we can obtain (after some calculations):

$$\frac{\partial m_1}{\partial \gamma} = \frac{\pi\delta\Delta(b-a\overline{q})(1-\delta\Delta)(a+\pi\varphi)\frac{\partial m_2}{\partial \gamma}}{\left[(1-\delta\Delta)(a+\pi\varphi) + \pi m_2\delta[1-\delta\Delta(1-\pi)]\right]^2} > 0$$, where the inequality holds because $b - a\overline{q} > 0$ and $\frac{\partial m_2}{\partial \gamma} > 0$.

With $m_2 > 0$, $m_1 > 0$, $\frac{\partial m_2}{\partial \gamma} > 0$ and $\frac{\partial m_1}{\partial \gamma} > 0$, we know from expression (17) that $\frac{\partial \tilde{S}}{\partial \gamma} < 0$. That is, a larger γ will reduce the critical level of cumulative violation at which firms will choose to comply with the regulation.

Proposition 2. *A larger γ in the penalty scheme will reduce the critical level of cumulative violation record at which a firm's optimal violation is zero.*

So far, we have shown that, under the dynamic enforcement strategy, a firm's optimal level of violation decreases with the cumulative violation record and converges to zero as the cumulative violation approaches the critical level $\tilde{S}$. To analyze how a firm will behave after its cumulative violation record reaches such a critical level, we differentiate the case between $\Delta = 1$ and $\Delta < 1$, where $\Delta = 1-\rho$ indicates the decay in the violation history and ρ is the rate of decay.

$\Delta = 1$ ($\rho = 0$) represents the case where there is no decay in the firms' past violation, i.e., the historical violation record that the regulator keeps is irreversible. Hence, the firms whose cumulative violation reaches the critical level $\tilde{S}$ will comply for all future periods. With no future violation and no decay, these firms' cumulative violation will remain at the critical level, which becomes a steady state.

For the case with $\Delta < 1$, firms behave in a different way. Even though a firm complies in the period when its cumulative violation record reaches the critical level, its historical violation can decay and fall below the critical level in the next period, which implies that the firm may choose to violate again. Besides, one can expect that the increase of the decay rate would increase the incentive for firms to violate, which will be verified through numerical simulations in Section 5.

4. Comparisons of the Two Enforcement Strategies

In this section, we compare firms' behavior under the dynamic enforcement strategy with that under the static enforcement strategy in terms of noncompliance level and its corresponding penalties. For the dynamic enforcement setting, we focus on the case where the decay rate in firms' noncompliance history is zero ($\Delta = 1$) due to its analytical tractability, implying that the cumulative noncompliance record is irreversible and that the regulator never forgets the violations recorded in the past. Recalling Section 3, with zero decay ($\Delta = 1$), there will be a steady state where firms will always comply.

4.1 Optimal Level of Noncompliance

Concerning the optimal violation amount, we compare (4) and (9), the first order conditions in the static and dynamic enforcement strategies. By setting the penalty rates for current noncompliance to be the same with $\phi = \varphi$, and thus $\Phi'(\cdot) = \Psi'(\cdot)$, the optimal level of violation would be lower in the dynamic enforcement strategy, because $M'(\cdot) \geq 0$ (recall that we have the coefficients of $m_2 > 0$ and $m_1 > 0$ in $M(S) = m_0 + m_1 S + \frac{1}{2} m_2 S^2$). This leads to the following proposition.

Proposition 3*. With the same penalty rates for current violation ($\phi = \varphi$), the optimal level of violation in the dynamic enforcement strategy is not higher than in the repeated static enforcement strategy.*

Now that we have compared the optimal noncompliance in each period, we proceed to compare total violations (the sum of violations in all the periods) in the dynamic enforcement strategy with total violations in the static strategy. As proven in Proposition 1, under the dynamic enforcement strategy, a firm's optimal level of noncompliance is decreasing in its violation record, which reduces to zero as the cumulative violation record approaches the critical level. For the traditional static enforcement strategy, the optimal violation is the same for each period. Given that we consider the incomplete compliance case only, the firm will commit the same level of violations in all periods. Therefore, in the long run, the dynamic enforcement strategy shows its advantage in reducing total violations, and the overall level of violations under a system of cumulative records is lower than that under static enforcement. This is summarized in the following proposition.

Proposition 4. *In the long run, the dynamic enforcement strategy results in a lower total violation than does the static enforcement strategy.*

4.2 Penalties in the Long Run

In terms of penalties, we compare the steady state level of penalty payment once caught under the dynamic enforcement strategy, which is equal to:

$$P^{\infty} = \frac{\varphi}{2}(V^{\infty})^2 + \frac{\gamma}{2}(S^{\infty})^2 = \frac{\gamma}{2}\left[\frac{b - a\bar{q} - \delta\pi m_1}{\delta\pi m_2 \Delta}\right]^2 \text{ because } V^{\infty} = 0$$

The derivative of P^{∞} w.r.t. the penalty rate on historical violation γ is calculated as:

$$\frac{\partial P^{\infty}}{\partial\gamma} = \left[\frac{b - a\bar{q} - \delta\pi m_1}{\delta\pi m_2 \Delta}\right]\left[\frac{(b - a\bar{q} - \delta\pi m_1)[m_2 - 2\gamma(\partial m_2 / \partial\gamma)] - 2\delta\pi m_2\gamma(\partial m_1 / \partial\gamma)}{2\delta\pi(m_2)^2\Delta}\right] \quad (21)$$

To judge the sign of $\frac{\partial P^{\infty}}{\partial\gamma}$, one first needs to know the sign of $b - a\bar{q} - \delta\pi m_1$. Note that (16.2) can be rewritten as:

$$m_1 = \frac{\pi m_2\delta\Delta[b - a\bar{q}]}{(1-\delta\Delta)(a+\pi\varphi) + \pi\delta m_2(1-\delta\Delta) + \delta\pi\cdot\pi m_2\delta\Delta} < \frac{\pi m_2\delta\Delta[b - a\bar{q}]}{\delta\pi\cdot\pi m_2\delta\Delta} = \frac{b - a\bar{q}}{\delta\pi}, \text{ which}$$

implies $b - a\bar{q} - \delta\pi m_1 > 0$.

Therefore, given that we have $\frac{\partial m_1}{\partial\gamma} = \frac{\pi\delta\Delta(b - a\bar{q})(1-\delta\Delta)(a+\pi\varphi)\frac{\partial m_2}{\partial\gamma}}{\left[(1-\delta\Delta)(a+\pi\varphi) + \pi m_2\delta[1-\delta\Delta(1-\pi)]\right]^2} > 0$, one knows from (21) that a sufficient condition for $\frac{\partial P^{\infty}}{\partial\gamma} < 0$ would be $m_2 - 2\gamma\frac{\partial m_2}{\partial\gamma} < 0$. It is not difficult to find:

$$m_2 - 2\gamma\frac{\partial m_2}{\partial\gamma} = \frac{-\pi^2\gamma\delta - (1-\delta\Delta^2)(a+\pi\varphi) + \sqrt{\Gamma} - \frac{2\gamma\Lambda}{\sqrt{\Gamma}}}{2\pi\delta[1-\delta\Delta^2(1-\pi)]}$$

where $\Gamma = [(1-\delta\Delta^2)(a+\pi\varphi) - \pi^2\gamma\delta]^2 + 4\pi^2\delta\gamma[1-\delta\Delta^2(1-\pi)](a+\pi\varphi)$ and $\Lambda = 2\pi^2\delta[1-\delta\Delta^2(1-\pi)][a+\pi\varphi] - \pi^2\delta(1-\delta\Delta^2)(a+\pi\varphi) + \pi^2\delta\pi^2\gamma\delta$ are positive. Therefore, if $\sqrt{\Gamma} - \frac{2\gamma\Lambda}{\sqrt{\Gamma}} < 0$, i.e., $\Gamma < 2\gamma\Lambda$, we would have $m_2 - 2\gamma\frac{\partial m_2}{\partial \gamma}$ and thus $\frac{\partial P^\infty}{\partial \gamma} < 0$.

Actually, $\Gamma < 2\gamma\Lambda$ implies the following:

$$[(1-\delta\Delta^2)(a+\pi\varphi) - \pi^2\gamma\delta]^2 + 4\pi^2\delta\gamma[1-\delta\Delta^2(1-\pi)](a+\pi\varphi)$$
$$< 4\pi^2\delta\gamma[1-\delta\Delta^2(1-\pi)][a+\pi\varphi] - 2\pi^2\delta\gamma(1-\delta\Delta^2)(a+\pi\varphi) + 2(\pi^2\delta\gamma)^2$$

which is equivalent to: $(1-\delta\Delta^2)^2(a+\pi\varphi)^2 < (\pi^2\delta\gamma)^2$. That is, $\gamma > \frac{(1-\delta\Delta^2)(a+\pi\varphi)}{\pi^2\delta}$.

Therefore, a sufficient condition for $\frac{\partial P^\infty}{\partial \gamma}$ <0 is $\gamma > \frac{(1-\delta\Delta^2)(a+\pi\varphi)}{\pi^2\delta}$. In other words, with a sufficiently large penalty rate γ, the steady penalty payment is decreasing with the penalty rate. That is, the higher the penalty rate, the lower the resulting penalty payment.

Also, as $\gamma \to +\infty$, we have: $m_2 \to +\infty$, $m_1 \to \frac{\Delta(b - a\overline{q})}{1 - \delta\Delta + \delta\pi\Delta}$, which implies:

$K_0 = \frac{b - a\overline{q} - \delta\pi m_1}{a + \pi(\varphi + \delta m_2)} \to 0$ and $K_1 = \frac{\delta\pi m_2\Delta}{a + \pi(\varphi + \delta m_2)} \to \frac{\delta\pi\Delta}{\delta\pi} = \Delta$. That is, if the punishment is high enough, the firm tends to comply even with no violation record $S = 0$ due to the deterrence effect, resulting in no penalties, $P^\infty \to 0$.

On the other hand, once a firm is caught under the static enforcement strategy, the penalty payment is:

$$P^{ST} = \frac{\phi}{2}(V^*)^2 = \frac{\phi}{2}\left[\frac{b - a\overline{q} - \pi\phi}{a}\right]^2$$

The derivative w.r.t. the penalty parameter yields:

$$\frac{\partial P^{ST}}{\partial \phi} = [b - a\overline{q} - \pi\phi][\frac{b - a\overline{q} - \pi\phi}{2a^2} - \pi\phi]$$

If $\phi < \frac{b - a\overline{q}}{\pi(2a^2 + 1)}$, we have $\frac{b - a\overline{q} - \pi\phi}{2a^2} > \pi\phi$, and thus $\frac{\partial P^{ST}}{\partial \phi} > 0$. But at the same time, violation $b - a\overline{q} - \pi\phi$ should be non-negative, which requires $b - a\overline{q} - \pi\phi \geq 0$, i.e., $\phi < \frac{b - a\overline{q}}{\pi}$. Therefore, we have: $\frac{\partial P^{ST}}{\partial \phi} > 0$ for $\phi < \frac{b - a\overline{q}}{\pi(2a^2 + 1)}$, and $\frac{\partial P^{ST}}{\partial \phi} < 0$ for $\frac{b - a\overline{q}}{\pi(2a^2 + 1)} < \phi < \frac{b - a\overline{q}}{\pi}$.

Because there is an upper bound for ϕ to ensure a non-corner solution, there exists an interval for the penalty payment. However, for the dynamic strategy, as $\gamma \to +\infty$, we still have the corner solution. As we have shown that $\frac{\partial P^{\infty}}{\partial \gamma} < 0$ for a sufficiently large γ, the penalty payment can be lower than that in the static strategy with a sufficiently large γ. Hence, the comparison of penalties sums up as the following proposition.

Proposition 5. *For a sufficient large γ, the penalty payment in the dynamic enforcement strategy will be lower than that in the static enforcement strategy.*

Based on the above two dimensions of comparison outcome – the violation amount and the resulting penalties – for both the dynamic and static enforcement setting, the dynamic strategy succeeds by lowering violation levels in both the short and long term. Besides, a higher compliance status does not necessarily involve stricter enforcement; instead the penalty amount can be lower.

Note that the monitoring probabilities are the same for both the static and dynamic enforcement strategies, which implies that the enforcement cost due to inspections does not differ for the two strategies. Therefore, let us focus instead on the enforcement cost due to the imposition of sanctions. As is known, the size of penalties can be considered as an indicator of the enforcement cost because more efforts by regulators or courts are usually required to impose a larger penalty on firms and it is usually more difficult to collect larger fines in practice. The positive relationship between penalty size and enforcement cost is commonly presumed in the literature (see, e.g., Garoupa (1997), who assumes a linear relationship between fine and enforcement cost). Therefore, Proposition 5 also implies that, with proper design of the penalty parameter γ, the dynamic enforcement strategy can lead to a lower enforcement effort, compared with the traditional static enforcement strategy in the literature.

5. Numerical Illustrations

This section aims to numerically illustrate the theoretical predictions above. In the first place, we specify a set of parameters that can serve as a benchmark for the dynamic enforcement strategy, as summarized in Table 1. In the following, we start the analysis with a representative firm, then proceed with the simulation of the whole industry.

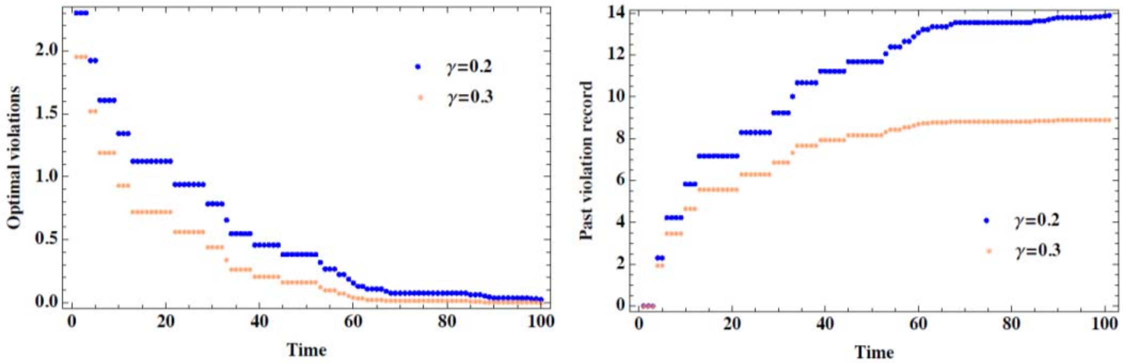
Table 1. Parameter Settings for Simulations of the Dynamic Enforcement Strategy

δ	a	b	f	φ	π	γ	ρ	$\bar{q}$
0.96	1	10	50	0.1	0.25	0.2	0	5

5.1 Simulation of a Representative Firm under the Dynamic Enforcement Strategy

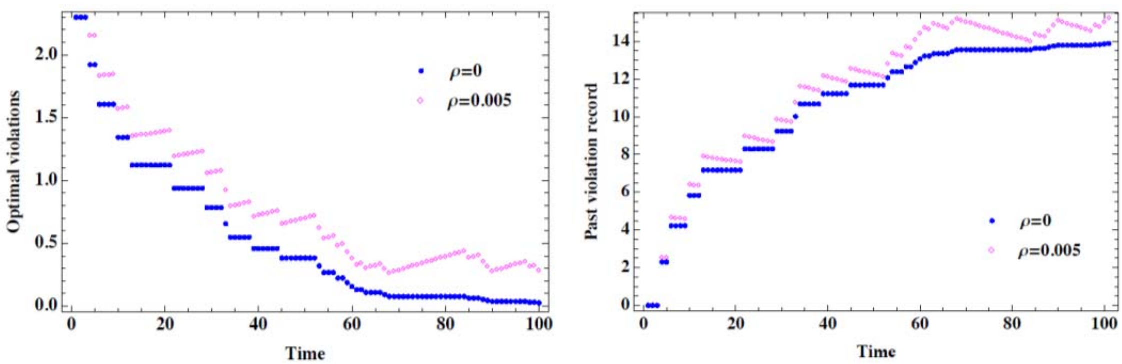
Figure 1 shows the optimal violation level of a single firm under the dynamic enforcement strategy. It can be seen that the optimal level of noncompliance is non-increasing and the noncompliance record is non-decreasing over time. If the firm is not inspected in one period, its cumulative violation does not change in the next period. As time passes, the optimal violation is converging to zero and the total violation is approaching a steady state level. Also, as the penalty rate becomes larger, the firm's optimal level of violation is reduced. These results are consistent with the theoretical predictions above.

Figure 1. Evolution of Violations and Noncompliance Record for a Firm under the Dynamic Enforcement Strategy



In the simulation presented in Figure 1, there is no decay in the firm's noncompliance record. To investigate the effect of decay on the firm's behavior, Figure 2 compares the results with no decay and those with a decay rate of $\rho = 0.005$. The optimal level of noncompliance under $\rho = 0.005$ is no longer non-increasing over time. This is due to the fact that, if the firm is not inspected for some periods, the decay in the noncompliance history results in a lower noncompliance record, which would give a firm the incentive to violate more. Accordingly, there is no longer a steady state for the violation record because, even if the violation record reaches such a steady level, the noncompliance record can decay and then the firm will start to violate again.

Figure 2. Effect of Decay Rate in Noncompliance Record in the Dynamic Enforcement Strategy

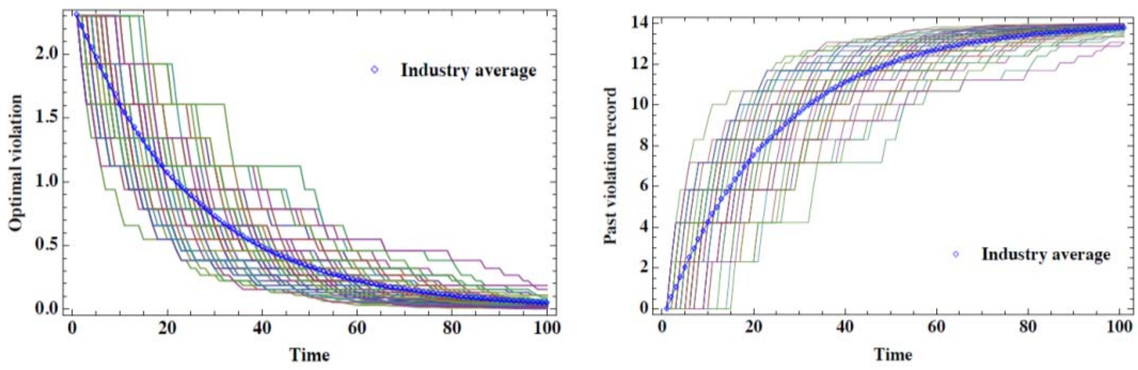


(a). Evolution of violations (b). Evolution of violation record

5.2 Simulation of the Whole Industry under the Dynamic Enforcement Strategy

Because the regulator randomly chooses which firms to inspect in each time period, the evolution of the noncompliance record is different for different firms. A simulation of the behavior of the whole industry is illustrated in Figure 3, where the total number of firms is set at $N = 60$, implying that, in each period, $n = N \times \pi = 15$ firms are randomly chosen for inspection. As a result, the industry average level of violations is decreasing and the average noncompliance record is increasing over time.

Figure 3. Evolution of Violations and Noncompliance Record for the Whole Industry



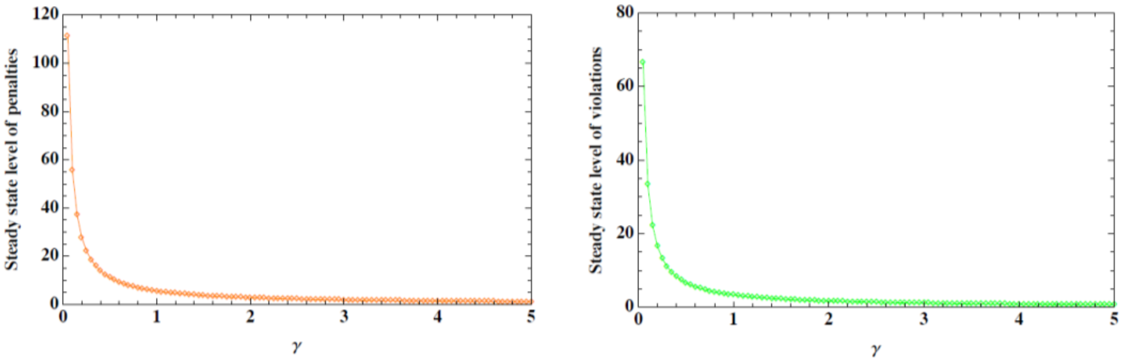
(a). Evolution of violations (b). Evolution of violation record

5.3 Comparisons of the Penalty for Different Enforcement Strategies

Figure 4 illustrates that the steady penalty in the dynamic enforcement strategy P^{∞} can be lower than the penalty payment under the static enforcement under some circumstances. In particular, the simulations show that the (steady) penalty payment in the dynamic enforcement strategy is decreasing with the penalty parameter γ, indicating a deterrent effect of the higher penalty parameter γ. Also, the steady level of violations tends to approach zero as γ becomes larger. Figure 5 shows that the penalty payment under the static enforcement strategy is not a monotonic function of the penalty parameter ϕ, even though optimal violation is monotonically decreasing with the strictness of the punishment.

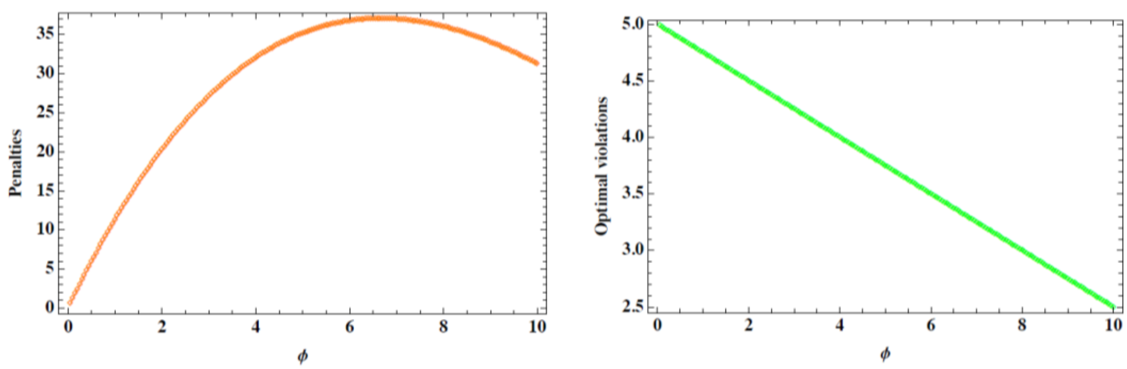
Comparing Figures 4 and Figure 5, we see that, as long as γ is set relatively large, the steady state level of penalty can be close to zero, which would of course be lower than the level under static enforcement.

Figure 4. Changes in the Steady State Level of Penalties and Violations in the Dynamic Enforcement Strategy



(a). Steady state level of penalties (b). Steady state level of violations

Figure 5. Changes in the Penalties and Violations in the Static Enforcement Strategy



(a). Penalties under the static enforcement strategy (b). Violations under the static enforcement strategy

6. Conclusion

This paper investigates firms' compliance behavior under a dynamic enforcement strategy where the penalties of violations depend on firms' cumulative noncompliance record in the past. Because the probability of being inspected is random, we develop a stochastic dynamic programming model to study firms' behavior under the dynamic enforcement strategy and compare their behavior under a traditional static enforcement strategy. We show that firms' optimal level of noncompliance would be a decreasing function of their past noncompliance record and that a more stringent enforcement strategy can reduce firms' expected fines due to the reduced violations. Comparisons with the repeated static enforcement strategy indicate that the dynamic enforcement strategy can be superior in terms of reducing violations and reducing the enforcement effort necessary to achieve a desired level of compliance (given that the size of fines is positively related to enforcement cost).

Note that the "dynamic" studied in this paper emphasizes the enforcement strategy where the penalty scheme depends on compliance history that evolves over time and firms look forward in a stochastic environment. This should be distinguished from the dynamic pollution model often explored in the literature. For instance, the dynamics studied in Hoel and Karp (2002), Requate (2005) and Winkler (2008) mainly focus on the stock accumulation of pollution. Besides, a recent work by Arguedas et al. (2014) studies noncompliance in a dynamic framework in which regulators interact with polluting firms and set pollution standards over time. They find a similar result: the noncompliance level decreases over time in the dynamic setting. In our dynamic enforcement mechanism, the optimal violation is decreasing and lower than that under static enforcement. Therefore, as time passes in our dynamic framework, lower enforcement efforts are needed to achieve the same (or higher) extent of compliance as under the traditional static enforcement. In other words, because the violation history of a polluting firm is easy to record and trace, our dynamic enforcement strategy is more cost-effective without reducing the compliance target. A possible direction for further research would be to investigate how our results would change in a game theoretical framework, where the interactions between the regulator and firms can be taken into account.

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