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The fossil endgame: strategic oil price discrimination and carbon taxation

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This paper analyzes how fossil fuel-producing countries can counteract climate policy. We analyze the exhaustion of oil resources and the subsequent transition to a backstop technology as a strategic game between the consumers and producers of oil, which we refer to simply as ‘OECD’ and ‘OPEC’, respectively. The consumers, OECD, derive benefits from oil, but worry about climate effects from carbon dioxide emissions. OECD has two instruments to manage this: it can tax fuel consumption and decide when to switch to a carbon-neutral backstop technology. The tax reduces climate damage and also appropriates some of the resource rent. OPEC retaliates by choosing a strategy of price discrimination, subsidizing oil in its domestic markets. The results show that price discrimination enables OPEC to avoid some of the adverse consequences of OECD’s fuel tax and its switch to the backstop technology by consuming a larger share of the oil in its own domestic markets. Our results suggest that persuading fossil exporters to stop subsidizing domestic consumption will be difficult.

Keywords: dynamic games; stock externalities; carbon tax; energy subsidies

1. Introduction

The focus of recent international climate negotiations has been burden-sharing by large or well-developed countries, such as the United States, China, EU member countries, India, Japan, South Africa, and Brazil, whose positions on it are far apart. Despite big differences in ambition and who they believe should pay, ultimately, these countries are likely to accept the notion that policy instruments are needed and that emitting carbon dioxide must become more expensive to consumers. To date, they have had a hard time agreeing at the UN Framework Convention on Climate Change (UNFCCC) Conference of the Parties meetings, and this has led many observers to look for emissions reduction policies that stop short of binding agreements. One of the policies promoted by the Group of 20 Finance Ministers and Central Bank Governors (G20) and the International Monetary Fund (IMF), for instance, is the abolition of fossil fuel subsidies, which appear all the more irrational because they raise emissions and are assumed to be bad for the global economy. According to the IMF (Coady \textit{et al.} 2010) and the International Energy Agency (IEA 2010), considerable economic benefits can be reaped by removing subsidies.\textsuperscript{1}

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The suppliers of fossil fuels tend to be strongly opposed to emissions taxes. They often argue that taxes do not reduce carbon emissions, but are merely a device by which importing governments steal the resource rent. We know that effective international agreement on climate policy is extremely tough to achieve. The fossil fuel-exporting countries are among those most likely to obstruct climate negotiations. It is therefore crucial to understand their economic motives, which is the heart of this paper.

The purpose of this paper is to analyze the strategic motives for oil-exporting countries to subsidize petroleum products in their domestic markets. This clearly risks encouraging domestic consumption, which undermines international efforts to reduce carbon emissions. Will this price differentiation reduce or eliminate the effects of carbon taxation?

The possibility of price differentiation between markets is potentially important because of the issue of carbon leakage. In this paper, we formally analyze the effect of price discrimination on domestic and international markets as a response to taxation by importing countries. We seek to understand the importance of these subsidy strategies to the efficiency of climate policy, and indirectly to its impact on international negotiations on climate change.

For an exhaustible resource that generates profits, efficient resource managers would ensure that the resource rent is reflected in the market price. Conventional wisdom suggests that this resource rent should rise exponentially, at least in simplified economic models (Hotelling 1931). However, a number of studies show that this pattern may change as a result of complex interactions between the taxation of externalities from fossil fuels and the scarcity rent (Sinclair 1992, 1994, Hoel 1993, Ulph and Ulph 1994, Wirl 1994, 1995, Tahvonen 1995, 1996, Hoel and Kverndokk 1996, Sinn 2007). Results from these studies show that the tax may decrease eventually as oil approaches depletion. For example, Sinclair (1992) concludes that constant taxes merely squeeze rents and have no impact on the time profile of extraction; expectations of falling energy taxes serve as the stimulus to reduce extraction rates and postpone the adverse consequences that carbon emissions induce. Carbon taxes not only serve the purpose of correcting externalities but may also enable countries importing oil to appropriate at least part of the resource rent (Santiago and Escriche 2001, Liski and Tahvonen 2004). Unfortunately, this fits very well with beliefs put forward by politicians in the oil-exporting countries: they are skeptical of the environmental zeal in the West and see the taxes as a means of snatching their rent.

This paper studies a non-cooperative open-loop Nash equilibrium carbon tax in a model with a strategic importer and a strategic exporter. We deviate from the previous literature by focusing on dual pricing decisions by the strategic exporter for its domestic and international markets. Empirical data show that many oil-producing countries discriminate in different markets, selling oil products more cheaply in the home market. We believe that this may be an important extension of the debate because earlier studies on emissions taxes have focused on international markets, ignoring the domestic markets of the exporters. However, a few studies have analyzed the importance of dual pricing for an oil-producing cartel that has got one domestic market where the cartel acts a social planner market and one international market where the cartel acts as monopolist; see Kalymon (1975), Brander and Djajic (1983) and Wirl (1983). These papers show that a cartel has an incentive to discriminate in pricing and set a lower price in the domestic markets.
The strategic motives of OPEC may in reality be very complex and include other incentives too (Cairns and Calfucura 2011). None of these papers take into account emissions taxation. Fischer and Laxminarayan (2004) studied a related set up where a monopolist faces two markets where they act as monopolists in both and where they may either price discriminate or not (depending on arbitrage possibilities between the markets). Given their assumptions on isoelastic demand functions they find that a monopolist facing two demand functions prefers a price path that is at least as steep as the socially efficient path.

OPEC’s domestic markets are important, consuming almost 20% of its annual primary oil extraction, and this share is expected to grow (Gately 2007). In some non-OPEC countries, such as Indonesia, the domestic market is close to one-half of its total annual oil output, and the net exports of some oil producers, such as Mexico, have fallen drastically because their domestic markets have grown so fast – which is partly a result of the low domestic price.

Our analysis uses a strategic game between the consumers and producers of oil. There are two agents: an energy resource-exporting cartel (hereafter OPEC, although it includes all oil exporters in this stylized model), which is assumed to be the only seller of fuel to importing countries, and a group of resource-importing countries (hereafter OECD, after the Organization for Economic Co-operation and Development). For the purposes of our game, we assume that both OPEC and OECD consume fuels produced from oil, and we assume that the oil is homogeneous and has only one possible substitute, referred to as the ‘backstop’. Clearly this is a vast simplification, removing gas, coal, and many other sources of energy from the picture, but it allows us to focus attention on important dynamics of the game.

We assume that OPEC is the sole producer and source of oil. OPEC thus faces the traditional dilemma of economizing with an exhaustible resource. OECD, on the other hand, is concerned about maximizing welfare, achieved (among other strategies) through the mitigation of environmental damage from the elevated atmospheric stock of carbon dioxide at the same time as OECD wants to use the oil. Naturally, this is again a simplification, but OPEC has, at least historically, been strongly against any regulation of emissions from oil. At its disposal, OECD has the possibility to tax oil.

If OECD taxes oil, OPEC might react strategically by increasing the producer price in order to receive a larger part of the tax revenues that would otherwise remain in the oil-importing countries. This would, however, lower demand and (beyond some point) revenues, and put the oil price beyond the full control of the fuel-exporting countries because the path of rent will also be affected by taxes levied by OECD. Our objective is to consider the optimal design of the carbon tax in the presence of a two-sided strategic interaction: the buyer can set and coordinate taxation, and understands the effect of taxes on fuel prices; and the seller can coordinate sales and understands the effect of sale prices on taxation.

We also analyze the optimal time path for OPEC’s oil extraction. We assume that OECD will switch to a carbon-neutral technology (the backstop), once the consumer price reaches the opportunity cost of the backstop. Therefore, the optimal time path implies overall depletion at a date we call $T$, and at a time $t^*$, when OECD stops importing and switches to the backstop technology. We particularly study the role of choke prices and the development of the backstop technology for the timing of resource depletion. The basic implications of backstop technologies for non-renewable resource markets have been analyzed in a range of papers (e.g. Nordhaus
In relation to climate change issues, the implications of backstops have been central to the discussion of the 'green paradox' in which policies such as a rising fuel tax may have the opposite effect to that intended (leading to an escalation of current exploitation; see Sinn 2007, Gerlagh 2011).

By way of comparison, our paper also considers a case without price discrimination, using just a uniform oil price that would be maintained in the absence of domestic subsidies in OPEC. This implies that domestic consumers have to pay a higher price for oil and, as a result, would consume less; thus, more oil can be exported and the oil lasts longer. This brings revenue to the exporters, but in the strategic game situation we model here, it also implies that OPEC has less ability to counteract the tax imposed by OECD and thus it also lowers OPEC’s net social payoff. This case also has direct policy relevance, in light of the G20’s call to reduce fossil fuel subsidies (IEA 2010).

In section 2, we describe the game model. In the third section we analyze the optimal taxation and the timing of depletion in a world with a uniform price (i.e. no price discrimination between the domestic market and the international market). The case with price discrimination between domestic and international markets is presented in section 4. These results allow us to see more clearly the effect of strategic gaming. The majority of the results are possible to derive analytically, but we also follow up with a simulation in section 5. The final section concludes, and the appendices contain the formal solutions to the game.

2. The model
To make the model as simple as possible, we assume that the world consists of two agents: one exporter, and one importer. For convenience, we refer to the resource-rich exporter as OPEC and the resource-poor importer as OECD, noting that they do not correspond exactly to the actual organizations. We assume that no oil is extracted within OECD. In reality, oil is extracted not only in the OPEC cartel countries, whose coalition sometimes suffers from stability problems, but also in a range of fringe countries, such as Russia, the United States, and Norway. However, for the purposes of our analysis, two agents will suffice.

The model includes two stocks – OPEC’s oil deposit stock $S$ in the ground and the carbon stock $E$ accumulating in the global atmosphere. Because carbon is transferred from OPEC’s oil deposit to the global atmosphere as a result of consumption in OECD and OPEC, we can express the change in stocks as a function of oil consumption in OPEC and OECD, hereafter indexed 1 and 2, respectively:

$$\frac{\partial S}{\partial t} = -(x_1 + x_2) \quad \text{and}$$

$$\frac{\partial E}{\partial t} = -\delta E + \gamma(x_1 + x_2),$$

where $x_1$ and $x_2$ are functions of time and, for simplicity, stand for $x_1(t)$ and $x_2(t)$. In fact all our variables are functions of time but including time indices will make the formulas unruly so they have been omitted throughout. Together they correspond to withdrawals from OPEC’s oil deposit stock $S$ in Equation (1) and add carbon to the
atmospheric stock $E$ at the transfer rate $\gamma > 0$ in Equation (2). $\delta$ is the decay rate of carbon dioxide from the atmosphere, crudely representing the net uptake of carbon dioxide by the biosphere and the oceans.

The annual oil consumption levels $x_1$ and $x_2$ are determined by OPEC, acting as a monopolist in the OECD market and as a social planner in the OPEC market, subject to the oil demand functions (3) and (4) in OPEC and OECD, respectively. For simplicity, we assume that the demand functions remain constant over time. The linear demand function on the OPEC market is given by:

$$x_1 = x_1 - \beta_1 \cdot \rho_1. \quad (3)$$

The demand function for OECD is also assumed to be linear up to a certain price, but beyond that it is assumed to be discontinuous, since it is assumed that OECD have access to a backstop technology:

$$x_2 = \begin{cases} x_2 - \beta_2 \cdot (p_2 + \tau) & \text{if } p_2 + \tau < \bar{p} \\ 0 & \text{if } p_2 + \tau \geq \bar{p} \end{cases}. \quad (4)$$

The backstop is a carbon-free source of energy that could be supplied at a cost equal to $\bar{p}$. This technology could be a simplified representation of options, such as carbon-neutral fuels or electricity generated from solar, wind, nuclear, or coal with carbon capture and storage. Here, we assume that $\bar{p} < x_2/\beta_2$; in other words, the backstop price is lower than the choke price when OECD demand falls to zero.

The policy instrument that OECD uses is a tax on oil consumption, $\tau$. The use of this tax will influence the time ($t^*$) at which a region stops using oil. There are three possible cases: (1) OECD stops using oil first and switches to the backstop, (2) OPEC stops first, and (3) OECD and OPEC stop simultaneously. We assume $x_1/\beta_1 > \bar{p}$; in other words, the maximum willingness to pay in OPEC is higher than $\bar{p}$. Note that consumers in OPEC will continue consuming oil even after OECD switches to the backstop. We focus our analysis on the first case.

3. Pricing strategies without price discrimination

We start with the assumption that OPEC is not able to discriminate on prices between its domestic market and the OECD market, implying that the producer price for the integrated domestic and international market is uniform. This assumption also implies there is no arbitrage opportunity between two markets.

3.1. Taxation strategy in OECD

The OECD social planner cares about OECD social welfare, covering its consumer surplus, its tax revenues, and the damage caused by carbon dioxide. Given OPEC’s uniform pricing strategy, OECD chooses its taxation strategy $\tau$, on OECD oil consumption. Thus, the OECD value function can be written as:

$$\int_0^T \left[ CS_2 + \tau \cdot x_2 - \theta E e^{-\rho_2 t} dt + \frac{CS_2}{\rho_2} e^{-\rho_2 T} - \int_0^T \theta E e^{-\rho_2 t} dt - \frac{\theta E_T}{\rho_2 + \delta} e^{-\rho_2 T} \right], \quad (5)$$
where $CS_2$ is the OECD consumer surplus from consuming oil; $\tau x_2$ is the tax revenue; $\theta \cdot E$ is the instantaneous damage from the stock of carbon; $\overline{CS_2e^{-\rho_2t}}/\rho_2$ is the consumer surplus from the backstop technology; and $\int_T^T \theta Ee^{-\rho_2t} dt + \theta E(T)e^{-\rho_2T}/(\rho_2 + \delta)$ is the damage caused by accumulated carbon emissions after time $t^*$, when OECD has switched to the backstop technology. (See Appendix 1 for the list of variables for further clarification.)

Using Equations (3) and (4), and integrating the scrap value function by parts, the problem for OECD can be formulated as:

$$
\int_0^t [(z_2 - \beta_2 \cdot (p + \tau))^2/(2\beta_2) + \tau (z_2 - \beta_2 \cdot (p + \tau)) - \theta \cdot E]e^{-\rho_2t} dt + Ve^{-\rho_2t},
$$

subject to the dynamics in Equations (1) and (2), and the demand functions (3) and (4), and where the scrap value $V = (x_2 - \beta_2 \cdot p)^2/(2\beta_2) - [\theta E(t^*) - \gamma \int_S^{T_S} \theta \cdot e^{-\rho_2(t(s)-t)} ds]/(\rho_2 + \delta)$. Note also that after time $t^*$, $x_2 = 0$.

Solving the problem for OECD in Appendix 2, the tax is found to contain two terms, the Pigovian shadow cost of carbon from oil consumption $-\gamma \cdot \psi_2$, and the value of resource stock $\lambda_2$ from OECD’s perspective:

$$
\tau = -\psi_2 \cdot \gamma + \lambda_2.
$$

The evolution of the shadow cost of carbon is:

$$
\psi_2 = \psi_2(t^*)e^{-\rho_2(t^*-t)} - \int_t^{t^*} \theta e^{-\rho_2(t-s)} ds,
$$

which is equal to the present value of its marginal accumulated damage. Furthermore, the shadow value of the resource stock:

$$
\lambda_2(t) = \lambda_2(t^*)e^{-\rho_2(t^*-t)}
$$

is equal to the present value of resource rent at time $t^*$ when OPEC stops exporting to OECD.

Rearranging the first-order conditions in Equation (A.2) in Appendix 2 and substituting the shadow values $\lambda_2(t^*)$ and $\psi_2(t^*)$, evaluated at the end of the game – given by transversality conditions in Equations (A.7) and (A.8) also in Appendix 2 – yield the optimal tax levied by OECD:

$$
\tau = \frac{\theta \gamma}{\rho_2 + \delta} - \frac{\theta \gamma}{\rho_2 + \delta} e^{-\rho_2(t^*-t)}.
$$

The two terms on the right-hand side of Equation (10) represent the shadow cost of consuming each unit of oil $(-\gamma \cdot \psi_2)$ and the shadow value of the oil stock for OECD. The shadow value part is equal to the marginal damage of the emitted carbon, which is independent of the stock and time as a result of the linearity of the damage function.

With a higher pure rate of time preference, the shadow cost of carbon in Equation (10) will decrease because the future damage becomes smaller. The shadow value of the resource stock is negative for OECD because the remaining stock of oil
in OECD when it switches to the backstop will be consumed by OPEC. For OECD, it implies only climate damage and no benefits. Hence, there is a cost to leaving oil in the ground when switching to the backstop. Because the shadow value of the remaining oil underground is exactly equal to the damage caused by carbon emitted at the time when OECD gets close to the transition to backstop technology, the tax becomes zero. The sum of these shadow values gives the socially efficient tax levied on oil consumption in OECD. Hence, the optimal tax falls over time. This result is close to Hoel and Kverndokk (1996) and Sinclair (1992), who find that the social planner will set a decreasing carbon tax when the marginal damage of carbon starts falling.

The dynamic motives behind the optimal tax path in Equation (10) can also be compared to Liski and Tahvonen (2004), who show that the optimal tax level of an importer such as OECD is larger than the Pigovian tax when damage from accumulated carbon dioxide is low. If damage is zero, the only motive for the tax is to steal the rent from the exporter, and hence the tax acts as an import-tariff which is high in the beginning and falls over time as the resource is being extracted. However, the larger the damage is, the stronger the Pigovian incentive with an increasing tax level over time since damage is increasing. If damage is severe the tax is still increasing but less than in the Pigovian case (there is an import subsidy) as a result of the strategic response to the exporter’s incentive to delay extraction and receive a higher price initially. Still Liski and Tahvonen (2004) do not consider consumption within the exporter cartel.

The optimal decreasing tax in Equation (10) also reflects the leakage effect from the existence of several separate markets for oil. The OECD tax has two effects: it discourages local oil consumption and thereby encourages oil consumption in OPEC. A higher tax in the beginning implies an earlier transition to the backstop technology in OECD (hence, lower carbon emissions today by OECD), and more oil for OPEC to consume (hence, higher carbon emissions in the future by OPEC). This, however, has some environmental advantage for OECD because the earlier emissions to the atmosphere would have created a larger loss.

### 3.2. Pricing strategy in OPEC without price discrimination

The problem for OPEC now is to choose the same price $p$ of oil for both domestic and international markets. The objective function, in which social welfare is maximized and where the world oil price and date of depletion are the two decision variables, is:

$$
\int_0^T \left[ x_1^2(p)/(2\beta_1) + px_1(p) + px_2(p, \tau) \right] e^{-\rho t} dt,
$$

subject to the dynamics of the oil stock (Equation (1) and the carbon stock, Equation (2)), the non-negative constraints of $x_1$ and $x_2$, and the constraint that the consumer price in OECD is not larger than the opportunity cost of the backstop technology – that is, $p + \tau \leq \bar{p}$.

Because we have assumed that the choke price in OPEC $\alpha_1/\beta_1$ is larger than the opportunity cost of the backstop technology $\bar{p}$, consumers in OPEC will continue consuming oil after OECD switches to the backstop until the resource is depleted. Let $t^*$ denote the time at which OPEC stops exporting. After $t^*$, OPEC supplies only
its own market and the problem for OPEC in Equation (11) degenerates to maximizing the objective function:

$$\int_{t^*}^{T} \left[ x_1^2(p)/(2\beta_1) + p x_1(p) \right] e^{-\rho_1 t} dt,$$

subject to:

$$\dot{S} = -x_1 = -(\alpha_1 - \beta_1 \cdot p) \quad \text{and} \quad x_1 = \alpha_1 - \beta_1 \cdot p \geq 0.$$  \hfill (13)

In this optimization problem, the price charged to domestic consumers equals the shadow value of the resource to OPEC’s social planner, which grows exponentially at the rate $\rho_1$. The price grows until it reaches the choke price and all consumers stop consuming oil; therefore, the fuel price and resource rent after $t^*$ can be expressed as:

$$p = \lambda_1 = \frac{x_1}{\beta_1} e^{-\rho_1 (T - t)}$$

and the respective domestic sales are:

$$x_1 = \alpha_1 - \beta_1 p_1 = \alpha_1 (1 - e^{-\rho_1 (T - t)}).$$  \hfill (15)

To fully solve the problem in Equation (11), we need to describe the pricing, extraction, and taxation path when OPEC supplies both its domestic and the international markets. From Appendix 4, we find the uniform price:

$$p = (\lambda_1 \beta_1 + \lambda_1 \beta_2 + \alpha_2 - \beta_2 \tau)/(\beta_1 + 2\beta_2).$$  \hfill (16)

At $t^*$, OECD will stop consuming oil and switch to the backstop and OPEC, as a social welfare maximizer, will start to supply only its domestic market at price $p^*$. In addition, we assume that arbitrage between the $T - t^*$ and $t^* - t$ periods – that is, before and after the switch to the backstop technology in OECD – is not possible, and hence there cannot be a discontinuous jump in the international oil price at $t^*$. This implies that the optimal oil price follows the path:

$$p^* = \frac{\alpha_1 e^{-\rho_1 (T - t^*)}}{\beta_1} = (\lambda_1 \beta_1 + \lambda_1 \beta_2 + \alpha_2 - \beta_2 \tau^*)/(\beta_1 + 2\beta_2) = \bar{p}$$  \hfill (17)

and the resource rent during $[0, t^*]$ is:

$$\lambda_1 = \lambda_1^* e^{-\rho_1 (t^* - t)} = \frac{(\beta_1 + 2\beta_2) \bar{p} - \alpha_2}{\beta_1 + \beta_2} e^{-\rho_1 (t^* - t)}.$$  \hfill (18)

The difference in timing $T - t^*$ can be solved from Equation (17):

$$T - t^* = \frac{1}{\rho_1} \ln \frac{\alpha_1}{\beta_1 (\bar{p} - \tau^*)} = \frac{1}{\rho_1} \ln \frac{\alpha_1}{\beta_1 \bar{p}}.$$  \hfill (19)
The difference in exit timing from the two markets is now determined by the choke price in OPEC and the backstop technology in OECD. The higher the choke price or the cheaper the backstop, the larger the difference in exit timing is between the two markets. In summary, the prices charged to domestic consumers in OPEC and OECD are given, respectively, by Equations (14) and (16):

\[
p = \begin{cases} 
  ((\beta_1 + 2\beta_2)\bar{p} - \alpha_2)e^{-\rho_1(t^*-t)} + \alpha_2 - \beta_2 \bar{p} & \text{if } t \in [0, t^*] \\
  \alpha_1 e^{-\rho_1(T-t)} & \text{if } t \in (t^*, T]
\end{cases}
\]

(20)

The respective demand quantities are given by the demand functions and we can solve the optimal time for OPEC to deplete the resource from the exhaustible condition, that is:

\[
S_0 = \int_0^T [x_1 + x_2] dt = \int_0^T x_1 dt + \int_{t^*}^T [x_1 + x_2] dt.
\]

(21)

On the right-hand side, the equation is implied from the result that the resource will be consumed in both markets before \( t^* \) and will serve only the domestic market from \( t^* \) until depletion.

4. Taxation and pricing with price discrimination

Given the dynamics of oil and carbon stocks in Equations (1) and (2), and in the demand functions in Equations (3) and (4), we solve for the open-loop Nash equilibrium. Here, OPEC can set different prices in the OPEC and OECD markets, and we assume that arbitrage is difficult or even impossible due to strict border control and the ability of OPEC to segregate the market. We assume that OECD can still tax the oil consumption of OECD consumers; therefore, OECD may face a different price path as a result of OPEC’s discrimination pricing strategy. The problem set up for OECD in Equations (5)–(10) remains the same, with the exception of a change in notation from \( p \) to \( p_2 \) in the demand function of OECD and from \( p \) to \( p_1 \) in the demand function of OPEC.

4.1. Pricing strategies in OPEC with price discrimination

OPEC acts as a monopolist vis-à-vis OECD and as a social welfare maximizer in the OPEC market. Consequently, OPEC’s objective function adds up to an OPEC consumer surplus and an OPEC producer surplus of extracting oil for both OPEC and OECD markets. Facing the two demand functions (3) and (4), OPEC can discriminate in its pricing. Besides welfare maximization, one reason for pricing differently which we do not take into account in our modeling is that OPEC governments may want to buy political support. They may expect that cheap oil will lead to industrialization or they may feel pressured by local opinion (suspicion by the public of its own leaders) to share the rent with the common man (or motorist).
OPEC chooses the domestic price of oil $p_1$ and the international price of oil $p_2$, both of which affect the optimal timing $T$ for the depletion of resources, given the taxation imposed by OECD, when maximizing the following objective function:

$$
\int_{0}^{T} \left[ \frac{x_1^2(p_1)}{2b_1} + p_1x_1(p_1) + p_2x_2(p_2, \tau) \right]e^{-\rho_1 t} \, dt.
$$

(22)

This is subject to the dynamics in Equations (1) and (2), given the demand functions (3) and (4), and the non-negativity constraints of $x_1$ and $x_2$, where $x_1^2(p_1)/(2b_1)$ is the OPEC consumer surplus and $p_1x_1(p_1)$ and $p_2x_2(p_2, \tau)$ are producer surpluses from the OPEC and OECD markets, respectively.

Because the choke price in OPEC is higher than the opportunity cost of the backstop technology available in OECD, OPEC will exit from the OECD market before stopping sales in the domestic market, which thereafter continues until the oil stock is depleted. Hence, at some point in time $t^* < T$ before total depletion at $T$, OPEC stops exporting to OECD and OECD switches to backstop technology.

As the sole supplier to OECD’s market, OPEC will balance the benefits of additional sales with costs, in terms of a lower price on oil and the forgone availability of the resource in the future. From Equations (A.22)–(A.26) in Appendix 4, the producer price in OECD can be described by:

$$
p_2 = \begin{cases} 
\frac{1}{2} \left( \frac{x_2}{b_2} + \frac{x_1}{\beta_1} e^{-\rho_1(T-t)} \right) & t \in [0, t^*] \\
\bar{p} - \tau & t \in (t^*, T] 
\end{cases}.
$$

(23)

The optimal price is influenced by two terms: the first is the OPEC shadow value of the resource that drives the price to increase over time; and second is the tax levied by OECD. Substituting the price in Equation (23) into demand function (4), the equilibrium supply in the OECD market becomes:

$$
x_2 = x_2 - \beta_2(p_2 + \tau) = \begin{cases} 
\frac{1}{2} \left( x_2 - \beta_2 \tau - \frac{x_1\beta_2}{\beta_1} e^{-\rho_1(T-t)} \right) & t \in [0, t^*] \\
0 & t \in (t^*, T] 
\end{cases}.
$$

(24)

From the condition that $p_2(t^*) + \tau(t^*) = \bar{p}$, we can derive the difference in the timing of exit from the two markets $T - t^*$:

$$
T - t^* = \frac{1}{\rho_1} \ln \left( \frac{x_1/\beta_1}{(2\bar{p} - x_2/\beta_2 - \tau)} \right) = \frac{1}{\rho_1} \ln \left( \frac{x_1/\beta_1}{(2\bar{p} - x_2/\beta_2)} \right),
$$

(25)

where the right-hand side uses the conclusion from Equation (10); in other words, at $t^*$ when OECD stops importing, the endogenously chosen optimal tax $t^*$ is equal to 0.

The difference $T - t^*$ in the timing of exit from the two markets is determined by the choke prices in OPEC and OECD, and the backstop technology in OECD. The larger the choke price in OPEC or the lower the cost of the backstop
technology in OECD, the larger the difference in exit timing is between the two markets.

OPEC’s goal in extracting and selling oil in the domestic market is to balance the marginal benefits and user cost of extraction. This can also be viewed as selecting the optimal price path supported by the respective sales in each moment. The optimal oil price charged by OPEC in its own market $p_1$ equals the shadow value of oil $\lambda_1$ from OPEC’s perspective, which grows at the discount rate of $\rho_1$ until it reaches the choke price in the market at time $T$:

$$p_1 = \lambda_1 = \frac{\alpha_1}{\beta_1} e^{-\rho_1(T-t)}, \quad (26)$$

and the corresponding equilibrium sales can be expressed as:

$$x_1 = \alpha_1 - \beta_1 p_1 = \alpha_1 (1 - e^{-\rho_1(T-t)}), \quad (27)$$

which declines over time as the price rises.

The optimal time for OPEC to deplete the resource can then be obtained by solving the identity equation for the exhaustible resource:

$$S_0 = \int_0^T [x_1(p_1) + x_2(p_2)] dt = \int_0^T x_1(p_1) dt + \int_0^T x_2(p_2) dt. \quad (28)$$

The exhaustible resource equation reflects the important fact that the limited resource can be consumed at different times and in different markets, and OPEC has the ability to allocate its resource across time and markets.

Summarizing, we find that the tax determined by OECD is set to balance the loss of consumer surplus from consuming fuel, the tax income, and the benefits of reducing carbon emissions. The optimal fuel tax includes both a Pigovian tax and a strategic trade policy component. Given that OPEC and OECD have the same time preferences, the tax declines over time.

We find that the larger the choke price in OPEC or the cheaper the backstop technology in OECD, the larger the difference in timing between $t^*$ and $T$. Also, when the backstop technology is improved, it will induce the consumers in OECD to substitute it for fossil fuel earlier, but it will have the opposite effect on the timing of resource depletion in OPEC. This implies a paradoxical situation similar to what is analyzed in Sinn (2007). The producer understands that a cheaper backstop decreases the future value of oil and hence lowers the price of oil today, thus increasing current exports; this leads, in turn, to increased early carbon emissions and greater present value damage in OECD.

5. Simulation analysis

In both the discriminatory and uniform pricing cases, the resource rent and the timing of depletion and exit from the markets are jointly determined. This makes analytical results difficult to obtain in explicit formulae that can be compared. We therefore proceed by carrying out simulations of pricing and taxing strategies under different cases and test for the sensitivity of the result to changes in essential parameters.
5.1. Oil and carbon data

Table 1 presents the base parameters in our simulation analysis. OPEC’s proven conventional oil reserves were estimated to be about 935 billion barrels in 2005 (BP.com 2006). This constitutes about 75% of the total proven conventional oil reserves. These data concern only proven reserves – not the ultimately recoverable reserves that include reserve growth due to technological progress and new reserve findings, which would increase the extractable reserve estimates. For simplicity, we stick to the proven reserves. In addition, this study neglects large amounts of unconventional oil reserves (plus coal and other fossil energy resources, etc.).

The current atmospheric stock of carbon dioxide is about 215 billion metric tons of carbon above the pre-industrial level (Intergovernmental Panel on Climate Change 2007). Roughly, each barrel of oil contains 6.1 gigajoules ($10^9$ joules) of energy, which implies that OPEC’s reserves are about 5500 exajoules ($10^{18}$ joules). The carbon content of oil is about 0.02 kilograms per megajoule ($10^6$ joules), thus the total carbon stock in the oil reserves is roughly 110 gigatons of carbon.

The marginal damage cost of carbon dioxide emissions is subject to enormous uncertainties. These are uncertainties both in the climate system itself and in how ecosystems and social systems will be affected by changes in the climate. In addition, the effectiveness of adapting to climate changes is most uncertain. Economic estimates of the social cost of carbon dioxide emissions are usually in the range of US $50–$500 per ton of carbon (Fischer and Morgenstern 2003, Pearce 2003, Tol 2005). We assume that the damage cost of carbon dioxide in the atmosphere is $5 per ton of atmospheric carbon per year. Given a discount rate of 4% and an annual decay rate of carbon dioxide in the atmosphere of 1%, we get a total discounted social cost of an additional ton of carbon dioxide emissions to be $100 per ton of carbon.

Several potential alternatives to conventional oil are currently being discussed – ethanol, hydrogen, synthetic diesel from coal, and so on – and it is still not clear which alternatives will be prominent energy sources and energy carriers in the future. We assume carbon-neutral hydrogen produced from a carbon-neutral source (e.g. renewables, nuclear, or coal with carbon capture and storage) as a backstop. The main reason for benchmarking the cost of the backstop for this technology is that the future cost of hydrogen has been extensively assessed. Current (optimistic) estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total OPEC reserve (billion barrels)</td>
<td>$S_0 = 935$</td>
</tr>
<tr>
<td>Atmospheric carbon above the preindustrial level (gigatons)</td>
<td>$E_0 = 215$</td>
</tr>
<tr>
<td>Carbon transfer coefficient (ton/barrel)</td>
<td>$\Gamma = 0.122$</td>
</tr>
<tr>
<td>Marginal yearly damage of carbon ($/ton carbon)</td>
<td>$\Theta = 5$</td>
</tr>
<tr>
<td>OPEC demand coefficient</td>
<td>$z_1 = 3.00$</td>
</tr>
<tr>
<td>- (billion barrel)</td>
<td>$\beta_1 = 0.015$</td>
</tr>
<tr>
<td>OECD demand coefficient</td>
<td>$z_2 = 23.00$</td>
</tr>
<tr>
<td>- (billion barrel)</td>
<td>$\beta_2 = 0.115$</td>
</tr>
<tr>
<td>Cost of backstop ($/per barrel equivalent)</td>
<td>$\bar{p} = 150$</td>
</tr>
<tr>
<td>Decay rate of carbon dioxide</td>
<td>$\sigma = 0.01$</td>
</tr>
<tr>
<td>Discount rate in OPEC</td>
<td>$\rho_1 = 0.04$</td>
</tr>
<tr>
<td>Discount rate in OECD</td>
<td>$\rho_2 = 0.04$</td>
</tr>
</tbody>
</table>
of this technology when mature are about $100–$200 per barrel of oil equivalent (see e.g. Johansson et al. 2009). We assume that the opportunity cost of the backstop technology is about $150 per barrel.

We calibrate the demand functions in OECD and OPEC from various empirical studies and data. OECD’s demand elasticity varies across time, space, and specifications, and responds asymmetrically to price changes (Gately and Huntington 2002). We simply use the elasticity (–0.5) from the preferred specification in Gately and Huntington’s work. We assume that the demand in OPEC is less price elastic at –0.2, according to the most recent estimate for oil demand in the Middle East (Narayan and Smyth 2007). The demand equations are estimated by linearization around average price and quantity values for the last five years for each region. This gives demand parameters for the linear demand function of $\beta \approx 0.115$ and $\beta \approx 23.0$ for OECD, and $\beta \approx 0.015$ and $\beta \approx 3.0$ for OPEC.

We assume that the decay and discount rate are 1% and 4%, respectively. The former is a simple approximation of more complicated non-linear representations of global carbon cycles. 8

5.2. Simulating the fossil endgame

The shadow value of oil in the ground for OPEC starts low and grows exponentially at the pure rate of time preference until it reaches the choke price in the dual pricing case (price discrimination). It also increases exponentially in the uniform pricing case, with the exception of a jump in the shadow value when OECD leaves the market (see Figure 1).

The comparison shows that the shadow value of oil is higher in the dual pricing case than in the uniform pricing case prior to the jump in the shadow value that occurs when OECD leaves the market. After OECD has left the market the shadow value of oil becomes higher in the uniform pricing case than in the price discrimination case. Then the effect of the backstop technology and the tax to suppress the shadow value disappears in the uniform case. The value of oil jumps by around $40 per barrel and then resumes exponential growth until it reaches the choke price. This shows that the ability to discriminate is clearly a benefit to OPEC.

Figure 1. Shadow value of oil for OPEC.
If OPEC can discriminate in its prices, it will set a significantly lower domestic price, implicitly subsidizing domestic consumers. This price is equal to the shadow value of oil shown in Figure 1. For the OECD market, OPEC would, however, set a higher price; Figure 2 shows both producer and consumer prices. The latter are, in both the uniform and the discrimination cases, equal to the producer price plus tax. Note that the rise in OECD prices is smaller than the fall in the domestic price in OPEC in the case where it discriminates in pricing, compared to the uniform pricing case.

Comparing Figures 1 and 2, we see that the gap between the prices charged to OPEC and OECD consumers diminishes as OECD approaches the switch to the backstop because the shadow value of oil for OPEC will increase faster than the tax imposed by OECD. Irrespective of whether OPEC discriminates in its price, the tax path imposed by OECD starts high and decreases over time, reaching zero when OECD stops importing oil. As illustrated in Figure 3, when OPEC discriminates in pricing, OECD responds by imposing a smaller tax as a result of the higher producer

![Figure 2. Consumer and producer prices in OECD.](image2)

![Figure 3. Optimal tax path in OECD.](image3)
price charged by OPEC. Note that, in this sense, OPEC is very successful. Far from being able to retaliate by raising the tax that OPEC dislikes so much, OECD actually lowers it. Price discrimination is doubly attractive to oil producers: not only does it (maybe) serve some domestic goals but it is also an effective way of persuading OECD to lower its taxes.

Figure 4 shows the corresponding paths for the sale of oil. The simulations show that the percentage of domestic consumption in OPEC is much higher with price discrimination than without it. And the market share of OPEC is about twice as high with the price discrimination strategy. The elimination of price discrimination by OPEC would smooth the path of total extraction, lowering it today and leaving more for the future. Corresponding to the extraction of oil, the cumulated emission trajectory also differs under the different pricing strategy by OPEC. Without the possibility to price discriminate, less extraction today implies smaller cumulated carbon in the atmosphere in the short to medium term – which, of course, has an environmental benefit. The switch to the backstop by OECD is delayed in the price discrimination case as compared to the uniform pricing case due to the different pricing strategies for consumer in OECD and OPEC with as a result different oil consumption paths; see Figure 4.

6. Conclusion
This paper analyzes oil taxation (by oil importers, played by OECD) as an instrument of climate policy in the context of a game, in which oil exporters (played by OPEC) subsidize local demand strategically to counteract taxation. The analytical model contributes to a better understanding of the strategic incentives among oil-producing and oil-consuming countries. The tax imposed by OECD contains both Pigovian and strategic elements. The first element equals the shadow cost of carbon emissions from each unit of oil consumption, and hence the accumulated damage. The strategic element starts high and falls monotonically, leading to a net zero tax when OECD leaves the market. By this tax profile, OECD counteracts the rising resource rent and postpones the consumption of oil to slow down emissions,
and thus reduce (discounted) climate damage. The incentives are further motivated by the fact that the oil that is left in the ground when OECD leaves the oil market will later be used by consumers in OPEC, which will cause environmental damage to OECD, but not lead to any benefits to them.

The main contribution of this paper is to introduce the possibility of price discrimination by oil producers. When possible, OPEC will sell oil more cheaply to its domestic market than to export markets. This is important because such discrimination is widely observed among oil-producing countries and has already led to substantial increases in domestic petroleum consumption by these countries. The discriminatory pricing strategy by OPEC has important consequences. It enables OPEC to reduce the ‘adverse’ consequence it perceives from the tax imposed by OECD, allowing it to recapture some of the rents it would lose through such taxation and even force OECD to lower its taxes. In this sense, exporters can retaliate against OECD taxation – and, in the real world, maybe even attract OECD industries – thereby reclaiming rents and undoing the climate policy of OECD.

Significantly lowering the domestic price of oil products may not be a good overall industrialization strategy, but it could be effective in attracting some of the petrochemical, plastics, fertilizer, and other industries that use oil or fossil fuel energy intensively. It makes the domestic market increasingly important as the resource stock is extracted toward depletion.

The climate effect of OECD taxation is also weakened because price discrimination increases the share of domestic (OPEC) consumption – a form of carbon leakage. This is hugely relevant for political discussions, which are typically framed in terms of compensation of lost export revenues for oil-exporting countries as a result of climate policies. Reducing domestic petroleum product prices is also a popular way to distribute rents in many countries that lack other more sophisticated distribution mechanisms. What is central from a climate change perspective is the fact that price discrimination actually leads to the conclusion that it is optimal for importing countries to lower their taxes below the Pigovian level.

Our results show that the efficacy of OECD carbon taxation is limited by the power of OPEC countries to discriminate in their prices. Sometimes subsidy reduction is portrayed as an easy ‘win–win’ strategy. This is a potentially dangerous underestimation of the forces at play. Price discrimination and local subsidies are definitely in the material interests of exporters and is also easy to defend ideologically because exporters argue that the main motive for taxation by importing countries is really to steal their rents.

The non-cooperative outcome of OPEC’s discriminatory pricing behavior will increase current carbon emissions and lead to faster depletion of resources and larger climate damage in the near term. The central option open to environmentally concerned policymakers in OECD is to negotiate with OPEC. They need to persuade the leaders of fossil fuel-exporting countries that climate change is a real concern. They also perhaps need to find ways to ensure that the producers do not stand to lose substantial oil rents from universal climate policies. OECD should want OPEC not to discriminate in its pricing but, ideally, to participate in taxing oil products. From a strategic game viewpoint, it seems that there may indeed be a case for finding some way to compensate the producers – for instance by transferring backstop technology or being generous when distributing emissions allowances to the energy exporting countries.
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Notes

1. For a World Bank analysis along similar lines, see Larsen and Shah (1992).
2. In December 2007, when international bulk prices for gasoline in Rotterdam were US $105/gallon, the retail consumer prices in some oil-producing countries were as follows: Iran, 18.4¢/gallon; Libya, 19.8¢/gallon; Kuwait, 41.9¢/gallon; Qatar, 32.8¢/gallon; and Saudi Arabia, 22.2¢/gallon.
3. Most analysis on how OPEC behaves concludes that it is not behaving as a textbook cartel, but that it still has an important influence on the world oil market price; see for example Smith (2005).
4. The choke price is the minimum price that brings down the demand in an area to zero.
5. Some major oil exporters, such as Norway, do not engage in price discrimination.
6. We assume that OPEC has less technology and easier access to oil, and therefore does not develop any backstop technology.
7. A tax levied on agents or market activities that pollute (adversely affect) the environment or that generate negative externalities.
8. For more in-depth studies of the carbon cycle, a multitude of different time constants are needed to reflect the different time scales at which carbon dioxide equilibrates between atmosphere, oceans, biomass, soil, sediments, and rocks (Archer et al. 2009).

References

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For OECD’s dynamic optimization problem stated in Equations (1)–(6), the current-value Hamiltonian is:

\[ H_2 = \frac{a_2}{C_0}b_2 / C_1 p + t(\lambda_2 - \psi_2 \cdot \gamma) - \theta \cdot E \]

\[ = \frac{a_2}{C_0}b_2 / C_1 p + t(\lambda_2 - \psi_2 \cdot \gamma) + \psi_2 \cdot \gamma(x_1 - \beta_1 \cdot p + a_2 - \beta_2 \cdot (p + \tau)). \]  

(A.1)

Using Pontryagin’s maximum principle, the necessary conditions are:

\[ \frac{\partial H_2}{\partial \tau} = -\beta_2 \dot{x} + \beta_2(\lambda_2 - \psi_2 \cdot \gamma) = 0, \]

(A.2)

\[ \dot{\psi}_2 = \rho_2 \psi_2 + \theta, \quad \text{and} \]

(A.3)

\[ \dot{\lambda}_2 = \rho_2 \lambda_2. \]

(A.4)

Solving differential Equations (8) and (9) yields the shadow cost of carbon:

\[ \psi_2(t) = \psi_2(t^*) e^{-\rho_2(t - t^*)} - \int_t^{t^*} \theta e^{-\rho_3(\xi - t)} d\xi \]  

(A.5)

and

\[ \lambda_2(t) = \lambda_2(t^*) e^{-\rho_2(t - t^*)}. \]  

(A.6)
The necessary conditions for an optimal solution are:

\[ \psi_2(t^*) = \frac{\partial V}{\partial E} = -\theta/(\rho_2 + \delta), \]  
\[ \lambda_2(t^*) \geq \frac{\partial V}{\partial S} = -\theta_\tau/(\rho_2 + \delta), \]  
with \( \lambda_2(t^*) = -\theta_\tau/(\rho_2 + \delta) \) if \( S(t^*) > 0 \), and

\[ H_2^*|_{t=t^*} = \sup_{x} H_2(x, x^*)|_{t=t^*} = \rho_2 V - \frac{\partial V}{\partial t} = -\theta \cdot E(t^*). \]  

Appendix 3

For the time \( t \in (t^*, T) \), the current-value free time Hamiltonian function for the problem (12) with its constraint is:

\[ H_1 = x_1^2(p)/(2\beta_1) + px_1(p) - \lambda_1 x_1 + \mu_1 x_1. \]  

The necessary conditions for an optimal solution are:

\[ \frac{\partial H_1}{\partial p} = -\beta_1 p + \lambda_1 \beta_1 - \mu_1 \beta_1 = 0, \]  
\[ \mu_1 x_1 = 0(x_1 \geq 0 \text{ if } \mu_1 = 0; x_1 = 0 \text{ if } \mu_1 > 0), \]  
\[ \dot{\lambda}_1 = \rho_1 \lambda_1, \]  
\[ \dot{\lambda}_1(T) S_T = 0, S_T = 0 \text{ if } \dot{\lambda}_1(T) > 0 \text{ and } S_T > 0 \text{ if } \dot{\lambda}_1(T) = 0, \]  
and

\[ H_1^*|_{t=T} = \sup_{\rho_1 \geq 0} H_1(x_1)|_{t=T} = 0. \]  

From (A.13), we obtain:

\[ \dot{\lambda}_1 = \dot{\lambda}_1(T) e^{-p_1(T-t)}, \]  
and from (A.11), we get:

\[ x_1 = \dot{x}_1 - \beta_1 \dot{\lambda}_1(T) e^{-p_1(T-t)}. \]  

From (A.15), we can conclude that (A.12) is binding. Hence

\[ \dot{\lambda}_1(T) = \dot{x}_1/\beta_1. \]

Appendix 4

For \( t \in [0, t^*] \), the current value Hamiltonian function is:

\[ H_1 = x_1^2(p)/(2\beta_1) + px_1(p) + px_2(p, \tau) - \dot{\lambda}_1(x_1 + x_2) + \mu_1 x_1 + \mu_2 x_2 + \eta(\bar{p} - p - \tau). \]  

The necessary conditions for an optimal solution are:

\[ \frac{\partial H_1}{\partial p} = -\beta_1 p + \lambda_1 \beta_1 - \mu_1 \beta_1 - 2p \beta_2 + x_2 - \beta_2 \tau + \dot{\lambda}_1 \beta_2 - \mu_2 \beta_2 - \eta = 0. \]
From (A.21), we get:

$$
\dot{\lambda}_1 = \rho_1 \lambda_1,
$$

(A.21)

$$
\mu_1 x_1 = 0 (\mu_1 = 0, \text{ when } x_1 > 0; \mu > 0 \text{ when } x_1 = 0),
$$

(A.22)

$$
\mu_2 x_2 = 0 (\mu_1 = 0, \text{ when } x_2 > 0; \mu_1 > 0 \text{ when } x_2 = 0), \text{ and }
$$

(A.23)

$$
\eta (\bar{p} - p - \tau) = 0 (\eta = 0, \text{ when } \bar{p} - p - \tau > 0; \eta > 0 \text{ when } \bar{p} - p - \tau = 0).
$$

(A.24)

From (A.21), we get:

$$
\dot{\lambda}_1 = \lambda_1 (t^*) e^{-\rho_1 (t^* - t)}.
$$

(A.25)

Considering the interior solution from (A.20), we obtain:

$$
p = (\lambda_1 \beta_1 + \lambda_2 \beta_2 + x_2 - \beta_2 \tau)/(\beta_1 + 2 \beta_2).
$$

(A.26)

The demand for OPEC and OECD are:

$$
x_1 = x_1 - \beta_1 (\beta_1 + \beta_2) \lambda_1 e^{-\rho_1 (t^* - t)} + x_2 - \beta_2 \tau)/(\beta_1 + 2 \beta_2)
$$

(A.27)

and the equilibrium demand in OECD’s market is:

$$
x_2 = (x_1 - \lambda_1 \beta_2 e^{-\rho_1 (t^* - t)} - \beta_2 \tau)/(\beta_1 + 2 \beta_2).
$$

(A.28)

**Appendix 5**

The current-value Hamiltonian of the free endpoint problem (22) can be written as:

$$
H_1 = \chi_1^2 (p_1)/(2 \beta_1) + p_1 x_1 (p_1) + p_2 x_2 (p_2, \tau) - \lambda_1 (x_1 + x_2) + \mu_1 x_1 + \mu_2 x_2 + \eta (\bar{p} - p_2 - \tau).
$$

(A.29)

Using Pontryagin’s maximum principle, the necessary conditions are:

$$
\frac{\partial H_1}{\partial p_1} = -\beta_1 p_1 + \lambda_1 \beta_1 - \mu_1 \beta_1 = 0,
$$

(A.30)

$$
\frac{\partial H_1}{\partial p_2} = -2 p_2 \beta_2 + x_2 - \beta_2 \tau + \lambda_1 \beta_2 - \mu_1 \beta_2 - \eta = 0, \text{ and }
$$

(A.31)

$$
\dot{\lambda}_1 = \rho_1 \lambda_1;
$$

(A.32)

with the Lagrangian constraints:

$$
\mu_1 x_1 = 0 (\mu_1 = 0, \text{ when } x_1 > 0; \mu_1 > 0 \text{ when } x_1 = 0),
$$

(A.33)

$$
\mu_2 x_2 = 0 (\mu_1 = 0, \text{ when } x_2 > 0; \mu_1 > 0 \text{ when } x_2 = 0), \text{ and }
$$

(A.34)

$$
\eta (\bar{p} - p_2 - \tau) = 0 (\eta = 0, \text{ when } \bar{p} - p_2 - \tau > 0; \eta > 0 \text{ when } \bar{p} - p - \tau = 0);
$$

(A.35)

and the transversality conditions are:

$$
\lambda_1 (T) S_T = 0, \lambda_1 (T) = 0 \text{ if } S_T > 0 \text{ and } \lambda_1 (T) > 0 \text{ if } S_T = 0, \text{ and }
$$

(A.36)
The optimal prices set by OPEC in domestic and OECD markets are given by the first-order conditions (A.30) and (A.31). Rearranging yields gives:

\[ p_1 = \lambda_1 - \mu_1, \quad \text{and} \]

\[ p_2 = \frac{1}{2} \left( \frac{\alpha_2}{\beta_2} + \lambda_1 - \mu_2 - \eta \right). \]  

The resource rent is found from solving differential Equation (A.32), resulting in:

\[ \lambda_1 = \lambda_1(T) e^{-p_1(T-t)} . \]  

The Lagrangian constraints (A.33), (A.34), and (A.35) are conditions of complementary slackness. As the OECD consumer price \( p_2 + \tau \) reaches the backstop level \( \bar{p} \), OECD switches to the backstop technology and OECD demand for oil falls to zero by Equation (4) and constraint (A.34) binds. Simultaneously, constraint (A.35) then also binds. One conclusion implied by transversality condition (A.37) is that the OPEC supply to the domestic market \( x_1(T) \) at the terminal time \( T \) goes to 0.

Using (A.37), we can get \( x_1(T) = \alpha_1 - \beta_1 \lambda_1(T) = 0 \), which implies that \( \lambda_1(T) = \frac{\alpha_1}{\beta_1} \). Hence:

\[ \lambda_1 = \lambda_1(T) e^{-p_1(T-t)} = \frac{\alpha_1}{\beta_1} e^{-p_1(T-t)}. \]